

# Hydrodynamic properties of rotary-disc extractor. IV. Axial profiles of flow characteristics and calculation of concentration profile

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The dependence of  $Pe_c$ ,  $Pe_d$ ,  $Z$ ,  $K_x$ , and  $d_{32}$  on axial position in a laboratory rotary-disc extraction column was determined experimentally by using the system water—acetone—toluene. These relationships were expressed by means of simple correlation relations and used for the calculation of the concentration profile of the substance transferred in apparatus in order that the effect of axial profiles might be appraised. Furthermore, the suitability of using the diffusion model for the calculation of rotary-disc extractor is discussed.

Экспериментально были определены зависимости  $Pe_c$ ,  $Pe_d$ ,  $Z$ ,  $K_x$  и  $d_{32}$  от аксиального положения в лабораторной роторно-дисковой экстракционной колонне при использовании системы вода—ацетон—толуол. Эти зависимости были представлены в виде простых корреляционных соотношений и были использованы для расчета концентрационного профиля распределяющегося вещества в колонне, чтобы было оценено влияние аксиальных профилей на него. Обсуждалась также применимость диффузионной модели для расчета роторно-дискового экстрактора.

One of the models, used according to literature for the study of the properties of rotary-disc extractor, is the diffusion model [1]. The analytical solution of the equations of this model completed with boundary conditions enables us to calculate the concentration profile of the transferred substance in both phases in apparatus provided the dispersed phase behaves as the second continuous phase. Therefore, it is necessary to know the values of phase velocities  $U_x$  and  $U_y$ , the coefficients of longitudinal mixing  $\epsilon_x$  and  $\epsilon_y$ , characterizing the magnitude of turbulent diffusion in both phases, the coefficient of substance transfer  $K_x$ , the specific interfacial surface  $a$ , the equilibrium relationship  $y^* = f(x)$ , the length of apparatus  $h_c$ , and the entrance concentrations of phases. The solution assumes constant values of the velocities, of the product  $K_x a$  and the coefficients  $\epsilon$  in the whole apparatus and a linear form of the equilibrium relationship. Since  $h_c$  is a parameter of this solution, it may be also used for the calculation of the length of apparatus.

In reality, these assumptions are not fulfilled. The dispersed phase is only gradually dispersed in extraction column, owing to mechanical stirring, the larger drops split into smaller ones and their translational velocity decreases and, for this reason, the value of hold-up  $Z$  (defined as the ratio of the volume of dispersed phase to the volume of apparatus) increases with increasing distance from the entrance of phase. Because of this fact, the hydrodynamic character of the flow of both phases changes and the values of  $\varepsilon_x$ ,  $\varepsilon_y$ , and  $K_x$  as well. Thus, an axial change in  $a$  also appears because it holds

$$a = \frac{6Z}{d_{32}} \quad (1)$$

Under these conditions, the approximate solution of equations of the diffusion model put forward by Rod [2] appears to be more convenient. This solution is based on introducing the modified concentrations of phases expressing the state for an ideal piston flow and replacing the differentials by differences. This procedure provides the possibility of numerically calculating the concentration profile and the length of apparatus from the known values of velocities,  $K_x$ ,  $a$ ,  $\varepsilon_1$ ,  $x_F$ ,  $x_R$ , and  $y_S$  with satisfactory accuracy. If we know the length of apparatus, the calculation of the boundary concentration  $x_R$  is to be carried out by the method of approximation. This method enables us to respect the real variable character of some quantities and does not necessitate linear form of the equilibrium relationship. The only drawback of this method consists in that it is founded on an estimate of the magnitude of the concentration jump at the entrance of one of the phases into apparatus, that means on the estimate of a value, e.g.  $x_0$ . This concentration jump is due to longitudinal mixing and may be found in apparatus experimentally by investigating the concentration profile of the substance transferred in both phases. The whole process of calculation is dependent on the accuracy of that estimate, which has as a consequence that the concentration jump must be estimated accurate to 9 decimals if the concentration profile is to be expressed by 5 valid decimals. This circumstance makes the calculation rather tedious.

On the basis of this method of calculation, a programme of calculation was elaborated for a unit Hewlett—Packard 9100 B and used for reverse calculation of the concentration profiles in a laboratory rotary-disc extraction column. The development diagram of this programme resembles that one proposed for the solution of the stage model of extractor on the basis of an analogous approach [3]. The programme made possible the calculation for both constant and variable values of  $\varepsilon$ ,  $a$ , and  $K_x$ .

By using the concentration profile of transferred substance found experimentally in both phases and the Rod method, we are able to determine the values of the coefficients of longitudinal mixing and the product  $K_x a$  in the form of dimensionless criteria of  $Pe$  and  $No_x$  valid for the whole column. This determination is to be

performed by means of the equations presented in [1] and the equilibrium relationship. Similarly, we may also determine the local values of  $Pe_{xi}$  and  $Pe_{yi}$  in apparatus as mean values of these quantities in five equal length sections of the column into which it was divided according to Fig. 1. For this purpose, the relationships already employed and modified with respect to new boundary conditions were used

$$Pe_{xi} = \frac{U_x h_c}{\varepsilon_{xi}} = \frac{x_{i-1} - x_i}{\int_{z_{i-1}}^{z_i} (X - x) dz} \quad (2)$$

$$Pe_{yi} = \frac{U_y h_c}{\varepsilon_{yi}} = \frac{y_{i-1} - y_i}{\int_{z_{i-1}}^{z_i} (y - Y) dz} \quad (3)$$

The analogous local values of  $No_{xi}$  cannot be found by this method from concentration profiles because it holds

$$X_{i-1} - X_i = No_x \int_{z_{i-1}}^{z_i} (x - x^*) dz \quad (4)$$

and for the local value of  $No_{xi}$  it may be written

$$No_{xi} = \frac{K_x a_i h_c}{U_x} = \frac{X_{i-1} - X_i}{\int_{z_{i-1}}^{z_i} (x - x^*) dz} \quad (5)$$

The quantities  $X$ ,  $Y$  represent the modified concentrations of both phases for piston flow, *i.e.* the flow without longitudinal mixing. By comparing the last two equations, we obtain  $No_x = No_{xi}$ . Therefore, the product  $K_x a$  may be considered constant in the whole apparatus.

The local values of  $d_{32i}$  and  $Z_i$  were determined from the photographs of drops of the dispersed phase in working extraction column [4]. The values of  $d_{32}$  calculated

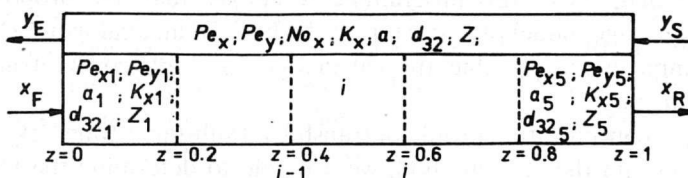


Fig. 1. Diagram of the division of column into five sections for calculating the axial profile of quantities.

for five positions photographed in the column were used for the calculation of the function  $d_{32} = f(z)$  from which the mean values of  $d_{32i}$  were calculated according to the following equation

$$d_{32i} = \frac{1}{0.2} \int_{z_{i-1}}^{z_i} f(z) dz \quad (6)$$

Assuming the real volume of the dispersed phase in the part of column represented in photograph  $V_d$  is related to the volume of dispersion  $\sum_i n_i d_i^3$ , which can be determined from the data on the number and size of drops, it holds

$$V_d = k \sum_i n_i d_i^3 \quad (7)$$

The constant  $k$  has equal value for all sections photographed in the scope of an experiment and the value of hold-up in individual parts  $Z'$  may be expressed in the form

$$Z' = \frac{V_d}{V_c} = \frac{k \sum_i n_i d_i^3}{V_c} \quad (8)$$

Hence, the mean value of hold-up in the whole column is given by the equation

$$Z = \int_0^1 Z' dz = k \int_0^1 \frac{\sum_i n_i d_i^3}{V_c} dz \quad (9)$$

The value of this integral may be estimated from the experimental course of subintegral function which, together with the known measured value of  $Z$ , is sufficient for calculating the constant  $k$  for each experiment as well as the local values  $Z'$  according to eqn (8). From the axial profile  $Z'$  thus determined,  $Z_i$  may be calculated according to the expression

$$Z_i = \frac{1}{0.2} \int_{z_{i-1}}^{z_i} Z' dz \quad (10)$$

Then the values of  $a_i$  may be calculated by means of eqn (1) and  $K_{xi}$  by means of the product  $K_x a$  obtained from the values of  $No_x$  and  $a_i$ .

### Experimental

The experimental determination of the concentration profiles of transferred substance, necessary for establishing the characteristics of two-phase flow, was carried out in a laboratory rotary-disc extraction column [4] with the system water—acetone—toluene

(depleted phase) the physical properties of which had been described in [2]. The technique of photographing the dispersion and the method of evaluating individual photographs is presented in [4]. Similarly, the experimental technique for taking samples of both phases for the determination of the concentration profiles and the method of calculation of the mean values of  $Pe_x$ ,  $Pe_y$ , and  $No_x$  for the whole apparatus are described in [1]. The values of phase velocities and the number of rotor revolutions at individual experiments are stated in [4]. By processing the experimental equilibrium data, the equilibrium relationship may be obtained in the form

$$y^* = 18.7 + 0.94 x \quad (11)$$

### Results and discussion

It appeared that the dependence of dimensionless hold-up  $\bar{Z} = Z'/Z$  on dimensionless simplex  $z/Z$  was, at a certain number of revolutions, linear with a constant section on the  $y$  axis and a slope which was a function of  $N$ . On ascertaining the form of this function, we obtained

$$\bar{Z} = \frac{Z'}{Z} = 9.34 \times 10^{-4} N^{1.7} \frac{z}{Z} + 0.79 \quad (12)$$

A confrontation of this expression with the values measured is shown in Fig. 2. This equation expressing the hold-up profile by means of dimensionless quantities depicts the form of this profile generally and makes possible a comparison with the hold-up profiles found in other extractors under different conditions. The constant

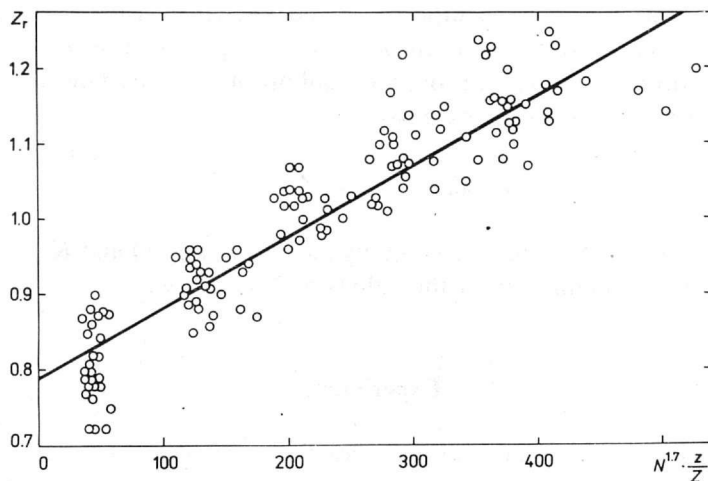


Fig. 2. Confrontation of the measured local values of hold-up with relation (12).

0.79 in eqn (12) gives the value of dimensionless hold-up at the entrance of the dispersed phase into apparatus and its value must depend on the way in which the on-coming dispersed phase splits, for instance, whether it enters the column through a distributor where it is dispersed, *etc.* This circumstance has not been investigated in more detail in this study. The dependence of the slope on  $N$  indicates that the form of the hold-up profile is determined by the amount of mechanical energy supplied by stirring owing to which the drops split, their surface energy rises, and thus, the hold-up increases.

The hold-up profile found is different from that presented by *Rod* [5]. This profile exhibits a maximum in the vicinity of the centre of apparatus. The difference is likely due to different effects influencing the hold-up in a laboratory column and in a plant for which the cited author determined the hold-up profiles.

The evaluation of the profiles of other quantities ( $Pe$ ,  $K_x$ ,  $d_{32}$ ), giving their relative values as functions of  $z$ , has shown that the most convenient is the exponential function

$$y = a \exp(b z) \quad (13)$$

because it is in best agreement with the measured values and gives nonzero values of  $y$  for  $z = 0$ . For the determination of the constants  $a$  and  $b$ , individual relations assume the form

$$\bar{d}_{32} = \frac{d'_{32}}{d_{32}} = 1.135 \exp(-0.26 z) \quad (14)$$

$$\bar{K}_x = \frac{K'_x}{K_x} = 1.45 \exp(-0.73 z) \quad (15)$$

$$\bar{Pe}_x = \frac{Pe'_x}{Pe_x} = 0.53 \exp(1.71 z) \quad (16)$$

$$\bar{Pe}_y = \frac{Pe'_y}{Pe_y} = 3.53 \exp(-2.21 z) \quad (17)$$

The quantities with index (') mean the local values at the distance  $z$  from the entrance of the depleted phase. All relationships have been derived on the assumption that their form does not depend on the ratio of velocities to  $N$  and a comparison of these values with the data of experiments 12—17 given in [4] is shown in Fig. 3.

The dependence of the mean diameter  $d_{32}$  on position could be expected on the basis of the experimental hold-up profile. The highest value of  $\bar{K}_x$  occurs at the entrance of the dispersed phase into apparatus. It decreases with increasing distance, which may be explained by that the substance transfer is more intensive in

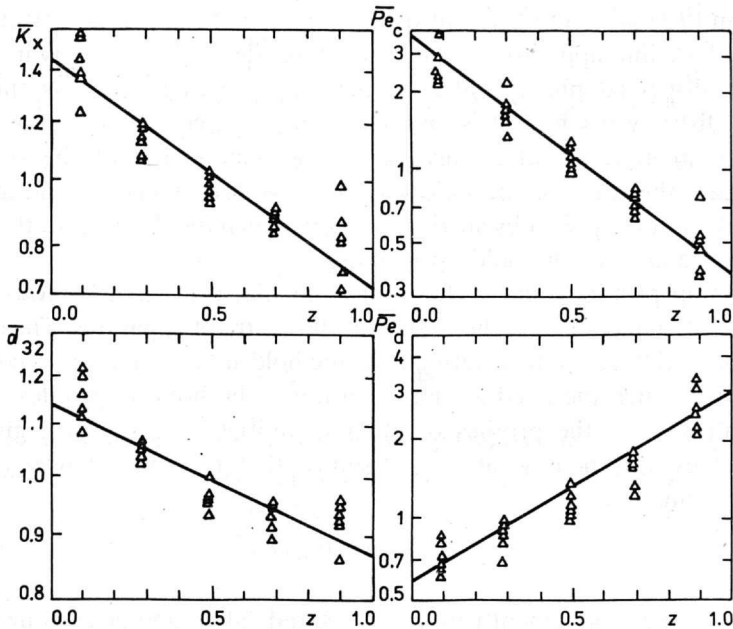


Fig. 3. Confrontation of the measured local values of quantities with relations (14–17) for experiments 12–17.

the regions where the drops are formed and split than in the regions where the number and size of drops is almost steady.

The value of  $\overline{Pe}_y$  decreases from the regions with lower hold-up to the regions with higher hold-up where there are more smaller drops with lower velocities and hence, with less intensive turbulence in the drifts behind them. Besides, the contribution of mechanical stirring also decreases in this direction [1]. The value of

$\overline{Pe}_x$  in the direction of dispersed phase flow also increases mainly owing to the increase in the contribution of mechanical stirring although the dispersion of drop residence time slightly decreases in this direction [6].

The programme of calculation, mentioned in introductory part, was so constructed that it respected or, if need, did not respect relations (12) and (14–16) and enabled us to calculate the local values of  $a_i$  by means of (1) for the determination of concentration profiles. Among the input data, the mean values of  $Pe_x$ ,  $Pe_y$ ,  $K_x$ ,  $Z$ , and  $d_{32}$  had to be given. With respect to the dimensionless distance  $z$ , used in the mentioned relations, it was also necessary, besides the estimate of  $x_0$ , to estimate the length of column  $h_c$  so long as this estimate was equal to the calculated value on the level of demanded accuracy. The programme was used for the calculation of profiles in each experiment on condition that a) the determining quantities were

constant (their experimental values were applied), *b*) the values of determining quantities varied along the column, *c*) the determining quantities were constant and linear equilibrium relationship (11) was used.

The profiles, thus calculated for each experiment, were compared with the measured values, as illustrated in Fig. 4 for experiment 15 presented in the table in

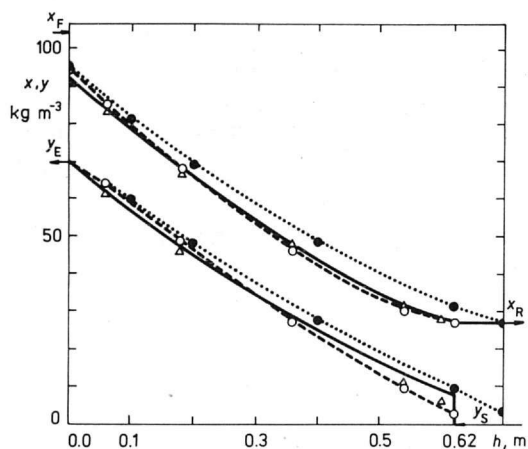


Fig. 4. Comparison of the measured concentration profile with the calculated concentration profiles. — Measured; ○ calculated by method a); △ calculated by method b); ● calculated by method c).

[4]. This comparison shows that the calculation according to *b*) is in the best agreement with experimental values. As a matter of fact, this calculation gives the values of  $h_c$  by 0.02 m smaller than the real values, but this difference may be neglected because it is equal to the length of the chosen step of calculation. The calculation according to *a*) differs from the preceding method very little, in particular, as regards the depleted phase. The difference is greater in the enriched phase and the form of the calculated profile is not consistent with that measured in the region of boundary values of  $h$ . The calculation according to *c*) gives the concentration profile and the length of column if a simplified linear equilibrium relationship is used. This profile differs from the measured one most. Thus, it may be concluded that the axial profiles of determining quantities so affect the values of the calculated concentrations that they accommodate the form of calculated profile to the form of measured profile.

It is worth noticing that a measured concentration profile does not, by its form, correspond to the boundary conditions of the diffusion model at the exits of phases, as formulated only intuitively by Danckwerts [7]



$$h = 0; \quad \frac{dy}{dh} = 0$$

$$h = h_c; \quad \frac{dx}{dh} = 0$$

(18)

which was also confirmed by all experiments. It seems that other boundary conditions should agree better with the real behaviour of extractor, for instance, the conditions put forward by *Wilburn* [8] who, in regard to situation in real plants, also took into consideration the influence of both sedimentation parts on concentration profile. In this case, the calculation, however, gets more complicated. Fig. 4 shows that, in spite of the differences between calculated and measured concentration profiles, the agreement of the real and calculated length of apparatus is very good, which confirms the suitability of using the diffusion model with original boundary conditions for designing.

### Symbols

$a$	specific interfacial surface
$d_i$	diameter of the $i$ -th drop
$d_{32}$	mean drop diameter of the order 3,2
	$d_{32} = \frac{\sum_i n_i d_i^3}{\sum_i n_i d_i^2}$
$h$	distance
$h_c$	length of column
$K$	total coefficient of substance transfer
$n_i$	number of drops with the diameter $d_i$
$No = \frac{K a h_c}{U}$	number of transfer units
$N$	number of rotor revolutions per time unit
$Pe = \frac{h_c U}{\varepsilon}$	Péclet number
$U$	velocity of phase
$V_d$	volume of dispersion
$V_c$	volume of the photographed part of column
$x$	concentration of depleted phase
$X$	concentration of depleted phase for piston flow
$y$	concentration of enriched phase

$Y$	concentration of enriched phase for piston flow
$z = \frac{h}{h_c}$	dimensionless distance
$Z$	hold-up of dispersed phase
$\varepsilon$	coefficient of longitudinal mixing

### Indices

E	exit of enriched phase
F	entrance of depleted phase from outside
H	entrance of enriched phase from inside
$i$	$i$ -th section of apparatus (Fig. 1)
O	entrance of depleted phase from inside
R	exit of depleted phase
S	entrance of enriched phase from outside
x	depleted phase
y	enriched phase

### Exponents

*	equilibrium state
	local value

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