

Mass Transfer in Mixing Processes

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Oxygen transfer in mixing process in equipment with mechanical mixing was searched. The tests were performed in a glass vessel of 30 dm³ liquid content with a loop mixer equipment. The obtained results have shown a good concordance with the values published in the literature, but at higher revolution frequencies the loop mixer exhibits better oxygen transport properties.

In the case when the oxygen electrode has a long time delay, a different procedure for the oxygen transfer coefficients evaluation was proposed and the results thus obtained are in a good conformity with the values calculated according to the method published in the literature. The measured values were elaborated into a nondimensional dependence.

In the chemical and food industries the mixing processes connected with mass transfer are often used. This situation is common in aerobic bioreactors (in submerged fermentation), where oxygen is transferred into a liquid phase (the continuum phase) and then into flocs of biomass. In such a case the mixer element serves not only for intensive content homogenization, but also for higher mass transfer, expressed by the overall volumetric mass transfer coefficient K_{1a} . This parameter is a very important indicator characterizing the bioreactor operation quality.

The mixing effect on the K_{1a} magnitude is expressed in the literature by the following relationship (see *e.g.* Refs. [1–6])

$$K_{1a} = B \left(\frac{P_g}{V_l} \right)^a j_g^b \quad (1)$$

Here one can object that eqn (1) is dimensionally inhomogeneous and therefore physically incorrect. Therefore we have proposed a relationship that takes into account the statement introduced in the previous sentence. The equation is as follows

$$Sh_1 \cdot ad_m = B Re_m^a N^b \quad (2)$$

where the criterion N was designed by ourselves and it is a member similar to the aeration intensity

$$N = \frac{Qd_m}{v_l D^2} \quad (3)$$

(see *e.g.* Refs. [2] and [7]).

Methods for Coefficient K_{1a} Estimation

These methods are described *e.g.* in Refs. [1] and [4]. Basically they can be divided into those performed in the real fermentation and in simulated conditions, respectively. We have used the gassing out method, which is an unstationary method in the simulated conditions – the oxygen transfer values were determined by changes of the oxygen concentrations in liquids. As a degassing medium a nitrogen was used. The process can be described mathematically by the following equation

$$\frac{dC_1}{dt} = \frac{K_{1a}}{1-\alpha} (C_1^* - C_1) \quad (4)$$

(see Ref. [8]), which after integration gives an equation of line, from which the K_{1a} values are determined

$$\ln(C_1^* - C_1) = \ln C_1^* - \frac{K_{1a}t}{1-\alpha} \quad (5)$$

Here it is assumed that the main resistance against the oxygen transfer is concentrated into the liquid phase.

Eqn (4) may be used when the oxygen electrode gives quick response. This electrode feature is expressed by the electrode time delay t_d (the time response) for reaching 90 % saturation of a solution with the oxygen at a sudden electrode movement from the solution with $C_1 = 0$ % into the solution with $C_1 = 100$ %. Very suitable for such experiments are membrane probes [9]. We used WTW oxygen electrodes – products of FRG, see Ref. [10]. The WTW manufacturer has declared for the electrode used in our experiments that $t_d \leq 10$ s. In this case a time electrode constant

t_e resulting from the electrode time response diagram is 3.2 s (Fig. 3). At low t_e values this parameter need not be considered, see *e.g.* Ref. [3]. In the opposite case another equation together with eqn (4) must be solved, which is as follows

$$\frac{dC_e}{dt} = \frac{C_1 - C_e}{t_e} \quad (6)$$

The common solution of both equations gives*

$$\frac{C_e}{C_1^*} = \left(1 - \frac{e^{-K_1 a t}}{1 - K_1 a t_e} + \frac{K_1 a e^{-\frac{t}{t_e}}}{\frac{1}{t_e} - K_1 a} \right) \quad (7)$$

EXPERIMENTAL

The measurements were performed in a glass container (300 mm i.d.) with a hemispheric bottom and the content volume 30 dm³. The mixing equipment was of a loop type (equipped with a deflector), impeller had three blades inclined by 45° to the horizontal plane (see STN 691025.3 and Fig. 1) and such an arrangement guaranteed a content motion with prevailing axial velocity. The air was fed through a rotameter to the annular distributor containing 40 openings with i.d. 0.8 mm each and was uniformly distributed through the vessel content.

During tests the revolution frequencies, torsion moments, air flow rates, temperatures, and oxygen concentration changes were measured and recorded. The revolution frequencies and the torsion moments were measured and displayed by a Steiger—Mohilo dynamometer (FRG product) with 0.01 Nm discerning ability. The dynamometer torsion element was located between two Hardy coupling parts (Fig. 2). The oxygen concentrations were taken off by the polarographic electrode EO 96, amplified and displayed by the microprocessor oximeter OXI 537 (both devices are products of WTW Co., FRG). The microprocessor oximeter measures also the content temperature and corrects the oxygen concentration data according to it. The oximeter signals were transmitted to the double-lines recorder TZ 500 (Verkon Co., Prague). The measuring procedure was as follows: At given n and air flow rate Q gassing out with the nitrogen was introduced. After reaching zero oxygen concentration, we transferred a three-way valve on to the aeration regime and then the response to this steep change was recorded. The measurement was stopped when 90 % saturation with oxygen was reached.

*Note: In eqn (4) the total content volume (the volume of the liquid + volume of the air) is considered. In our case only the liquid volume was taken into account and therefore the α values can be omitted. Moreover, in all our measurements they were under 1 %.

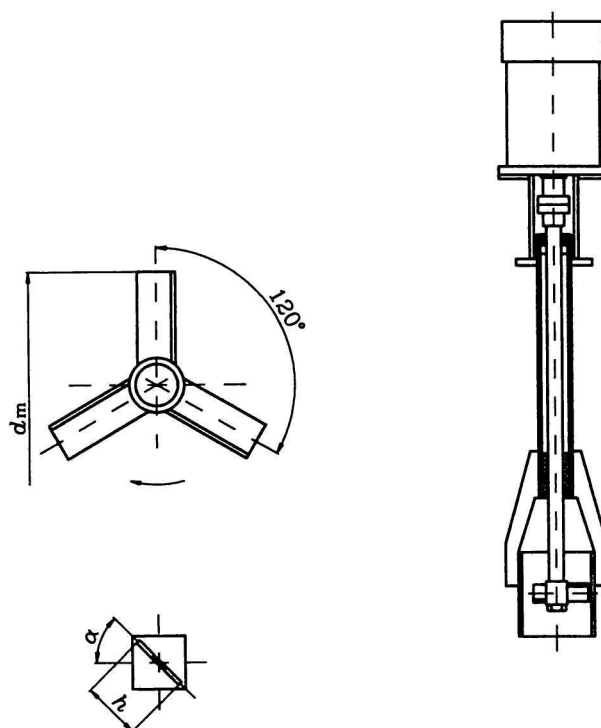


Fig. 1. Scheme of the experimental plant.

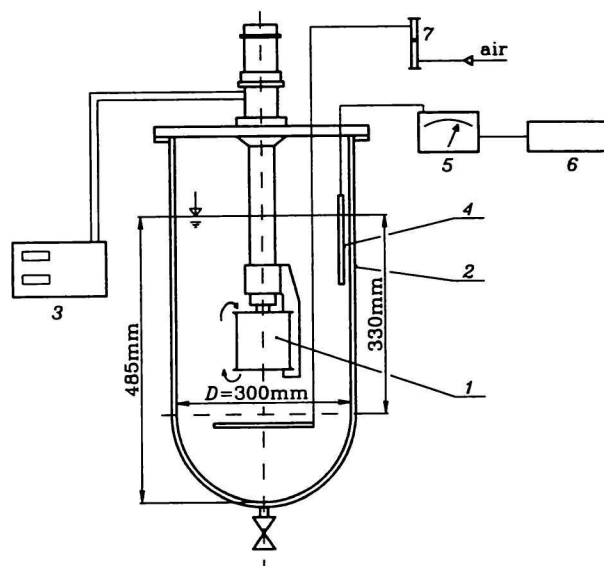


Fig. 2. The loop mixer arrangement. 1. Loop mixer, 2. glass vessel, 3. dynamometer, 4. oxygen electrode, 5. oximeter, 6. recorder, 7. rotameter.

During the tests two contents were used: tap water and saccharose—water solution. Sugar content was 20 °Bx (20 mass %). Properties of the both contents are introduced in Table 1 and were determined according to Ref. [11].

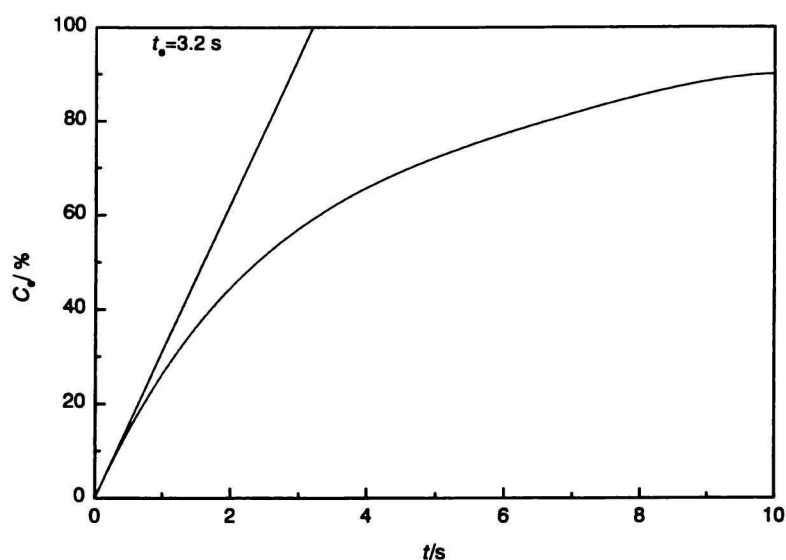
At first we started to measure with an electrode which had the time delay $t_d = 21$ s, which did not

Table 1. Properties of the Media Used in the Tests according to Ref. [11]

Medium	Density at 20°C	Kinematic viscosity at 20°C
	kg m ⁻³	m ² s ⁻¹
Water	998.2	1.0 × 10 ⁻⁶
Sugar solution 20 mass %	1081	1.82 × 10 ⁻⁶

Table 2. Comparison of Results of the Measurements of K_1a/s^{-1} at Different P_g for $Q = 0.58 \text{ dm}^3 \text{ s}^{-1}$

Source	2.6 W	4.6 W	7.6 W	12.8 W	25.9 W	40.6 W
This paper	0.015	0.019	0.021	0.028	0.039	0.052
Eqn (8)	0.014	0.018	0.022	0.027	0.035	0.042
Eqn (9)	0.012	0.016	0.020	0.026	0.036	0.045

**Fig. 3.** Time delay of the oxygen electrode.

correspond to the value declared by the producer. In this case the electrode time constant was above 7 s and eqn (7) was considered. For K_1a evaluations an iterative program in BASIC language was elaborated. Here it is necessary to mention that the third member in eqn (7) can be neglected due to its low value. Fig. 3 illustrates situation, when new electrode head was used. In this case the electrode time constant is 3.2 s and K_1a values may be calculated by eqn (6).

RESULTS AND DISCUSSION

Figs. 4 and 5 illustrate courses of K_1a values for the water content depending on the power measured by the dynamometer and the revolution frequencies, respectively. Figs. 6 and 7 demonstrate these values for the sugar—water solution (20 mass %). Comparisons of our results with the values calculated from the equations given by authors of papers [4] and [5] are in-

troducted in Table 2. The relationship for the oxygen transfer into the water content is as follows [4]

$$K_1a = 0.026 \left(\frac{P_g}{V_l} \right)^{0.4} j_g^{0.5} \quad (8)$$

and that in [5] for the same case

$$K_1a = 0.01 \left(\frac{P_g}{V_l} \right)^{0.475} j_g^{0.4} \quad (9)$$

Despite of a good concordance of our results with eqns (8) and (9) we must say that in these equations the limits of their validity are missing. That is, when $n = 0$, then for both mentioned equations K_1a must be zero. In this case the system acts as a bubble column and the oxygen transfer coefficients magnitudes are evident from Table 2. For such an operation regime the authors of paper [4] introduce the relationship

$$K_1a = 0.32 j_g^{0.7} \quad (10)$$

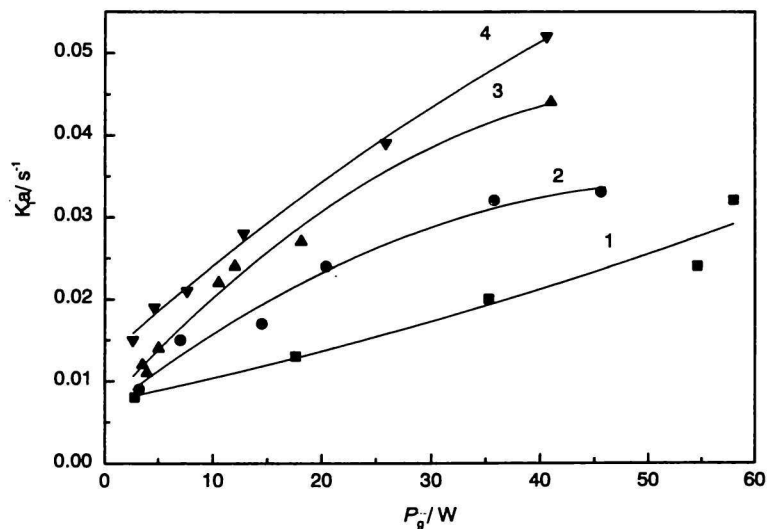


Fig. 4. Values of K_{1a} in dependence on P_g for the water content mixing. Parameter is air flow rate. 1. $Q = 0.13 \text{ dm}^3 \text{ s}^{-1}$, 2. $Q = 0.32 \text{ dm}^3 \text{ s}^{-1}$, 3. $Q = 0.40 \text{ dm}^3 \text{ s}^{-1}$, 4. $Q = 0.58 \text{ dm}^3 \text{ s}^{-1}$.

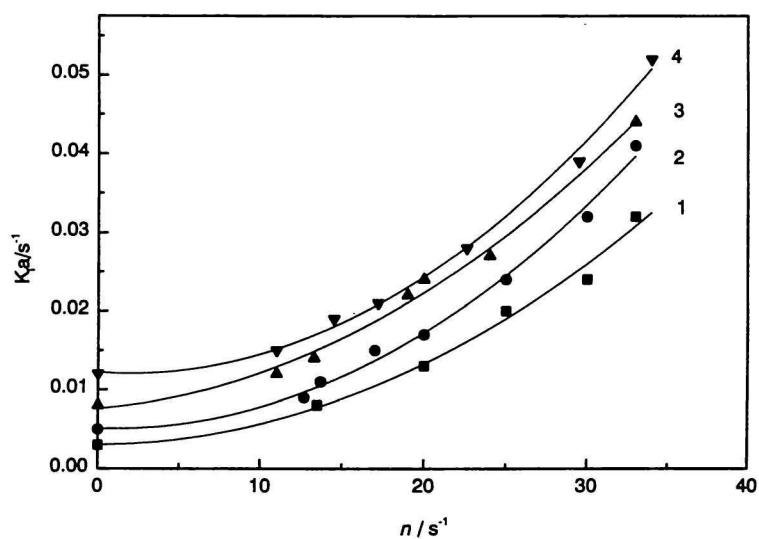


Fig. 5. Values of K_{1a} in dependence on n for the water content mixing. Parameter is air flow rate. 1. $Q = 0.13 \text{ dm}^3 \text{ s}^{-1}$, 2. $Q = 0.32 \text{ dm}^3 \text{ s}^{-1}$, 3. $Q = 0.40 \text{ dm}^3 \text{ s}^{-1}$, 4. $Q = 0.58 \text{ dm}^3 \text{ s}^{-1}$.

and the authors of paper [12]

$$K_{1a} = 0.39j_g^{0.67} \quad (11)$$

when zero concentration of suspended solids is considered, and finally according to [6]

$$Sh \cdot aD = 0.6Sc^{0.5} \left(\frac{gD^2\rho_l}{\sigma} \right)^{0.62} \left(\frac{gD^3}{\nu_l^2} \right)^{0.31} \alpha^{1.1} \quad (12)$$

Comparison of the values calculated according to eqns (10) and (11) with results of this work is in Table 3. Eqn (12) does not take into account the gas flux.

Judat [6] summarized several results of various researchers and for mechanically stirred reactor pro-

posed the following equation

$$(K_{1a})^* = 9.8 \times 10^{-5} \frac{\left(\frac{P_g}{V_1} \right)^{0.4}}{G^{-0.6} + 0.81 \times 10^{-\frac{0.65}{G}}} \quad (13)$$

The dimensionless parameters used in this equation are defined as follows

$$(K_{1a})^* = K_{1a} \left(\frac{\nu_l}{g^2} \right)^{\frac{1}{3}} \quad (14)$$

is the mass transfer parameter,

$$\left(\frac{P_g}{V_1} \right)^* = \frac{P_g}{\rho(\nu_l g^4)^{\frac{1}{3}}} \quad (15)$$

Table 3. Comparison of Results of the Measurements of K_{1a}/s^{-1} at Different Q (the system acts as the bubble column)

Source	$0.13 \text{ dm}^3 \text{ s}^{-1}$	$0.32 \text{ dm}^3 \text{ s}^{-1}$	$0.40 \text{ dm}^3 \text{ s}^{-1}$	$0.58 \text{ dm}^3 \text{ s}^{-1}$
This paper	0.003	0.005	0.007	0.012
Eqn (10)	0.004	0.007	0.008	0.011
Eqn (11)	0.006	0.011	0.012	0.016

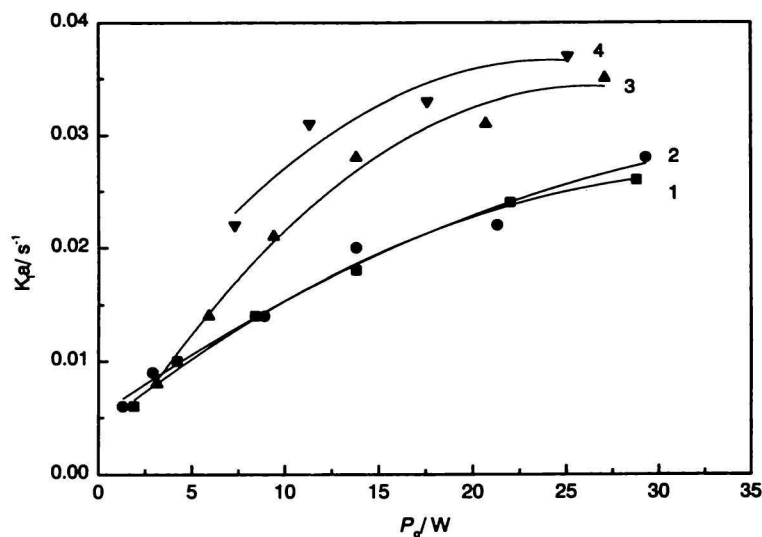


Fig. 6. Values of K_{1a} in dependence on P_g for the sugar solution mixing. Parameter is air flow rate. 1. $Q = 0.28 \text{ dm}^3 \text{ s}^{-1}$, 2. $Q = 0.32 \text{ dm}^3 \text{ s}^{-1}$, 3. $Q = 0.40 \text{ dm}^3 \text{ s}^{-1}$, 4. $Q = 0.53 \text{ dm}^3 \text{ s}^{-1}$.

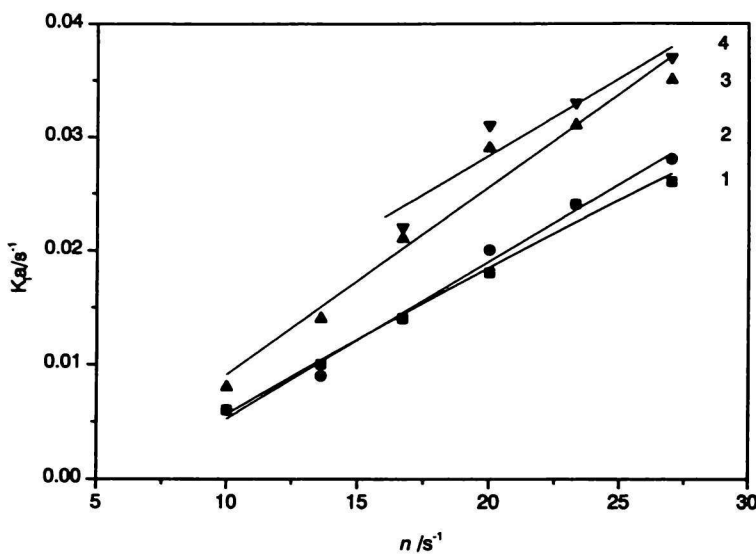


Fig. 7. Values of K_{1a} in dependence on n for the sugar solution mixing. Parameter is air flow rate. 1. $Q = 0.28 \text{ dm}^3 \text{ s}^{-1}$, 2. $Q = 0.32 \text{ dm}^3 \text{ s}^{-1}$, 3. $Q = 0.40 \text{ dm}^3 \text{ s}^{-1}$, 4. $Q = 0.53 \text{ dm}^3 \text{ s}^{-1}$.

the power parameter and finally,

$$G = \frac{Q}{\sqrt[3]{D^2 \nu g}} \quad (16)$$

is the aeration parameter.

From Table 4 one can see that at low n a good concordance between our results and the values calculated by eqn (13) exists. But the loop mixer gives

higher K_{1a} values at higher n . This fact is valid also for the values calculated by eqns (8) and (9). Moreover, eqn (13) satisfies the condition for the dimensional homogeneity.

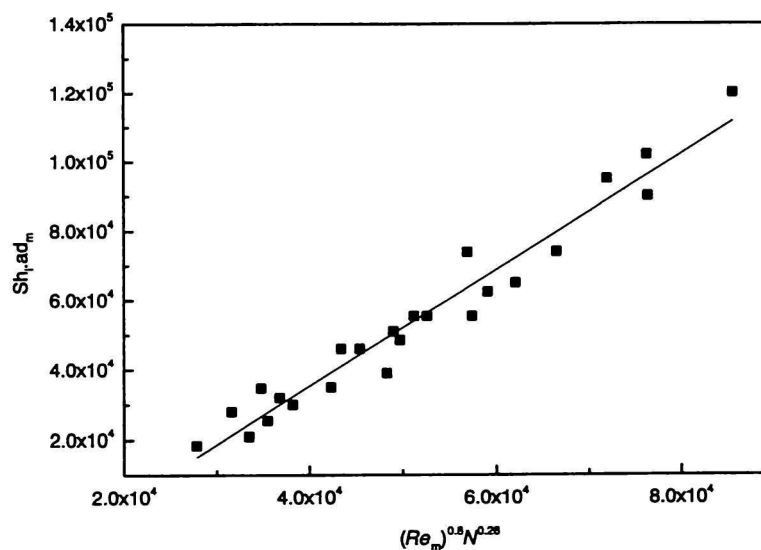
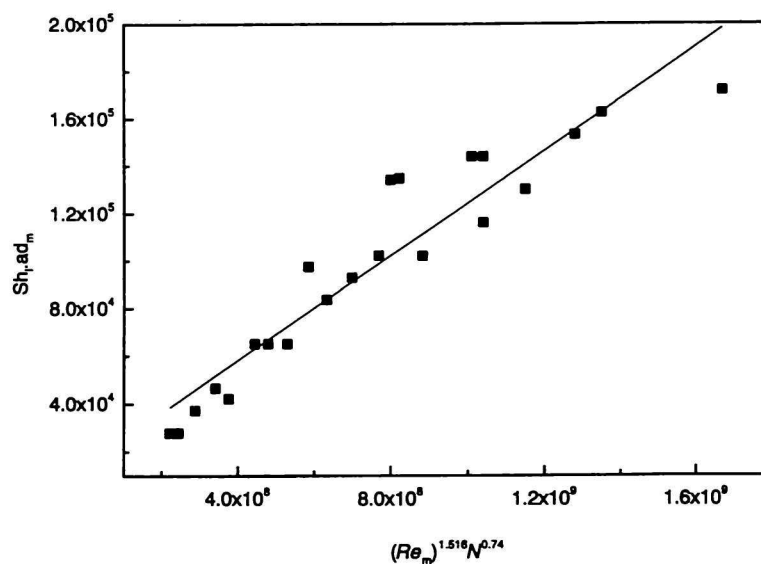
Authors of paper [13] introduce the following relationship

$$K_{1a} = 4.1 \times 10^{-3} \left(\frac{P}{V_1} \right)^{0.67} j_g^{0.31} \quad (17)$$

This equation considers instead of P_g the total power

Table 4. Comparison of Results of the Measurements of K_{1a}/s^{-1} at Different P_g for $Q = 0.40 \text{ dm}^3 \text{ s}^{-1}$

Source	3.5 W	5.0 W	9.5 W	10.1 W	18.1 W	45.6 W
This paper	0.012	0.014	0.022	0.024	0.027	0.044
Eqn (13)	0.012	0.014	0.017	0.018	0.021	0.033

**Fig. 8.** Nondimensional dependence for K_{1a} determination. Mixing of the water content.**Fig. 9.** Nondimensional dependence for K_{1a} determination. Mixing of the sugar solution.

input P (without gas) and the results calculated according to it are significantly higher compared to the measured values.

The performed tests pointed out good transport properties of the loop mixer. Besides the appropriate mixing properties and the low power numbers ($Po = 0.98$), the equipment is suitable also for technologies where content aeration is necessary. According to our design Bratislavské potravinárske strojárne manufactured such a mixer type with 3000 dm^3 working volume. The mixer has been used successfully in the

thick juice clarification process at the Dunajská Streda Sugar Factory. Taking into account the good results of this work we seek the opportunity to use the equipment in an industrial process connected with aeration.

The tests pointed out the following facts:

The results obtained by ourselves correspond very well within lower revolution frequencies with the values calculated according to eqns (8), (9), and (12).

Eqns (8) and (9) mentioned above do not consider the case when $n = 0$. In this case the system acts as the bubble column.

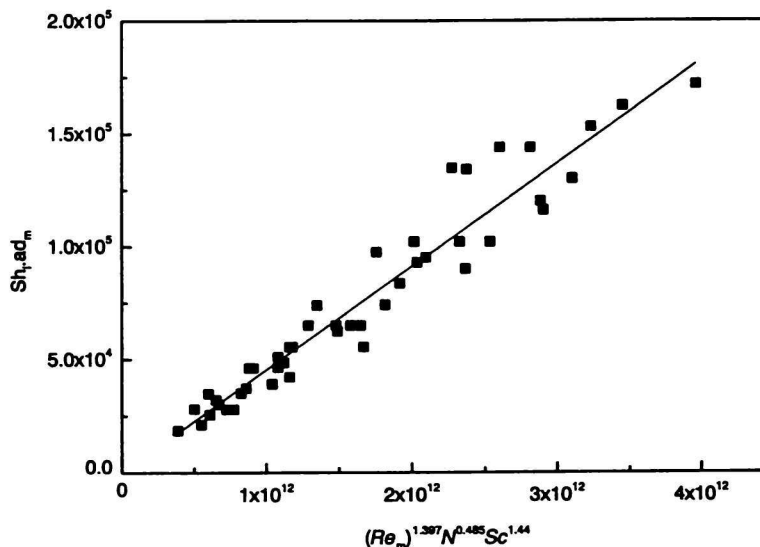


Fig. 10. Nondimensional dependence for K_{1a} determination. Mixing of the water and the sugar solution.

Majority of the relationships for the oxygen transfer coefficient evaluation are not dimensionally homogeneous, with the exception of (12) and (13). According to our statement given at the beginning of this paper the following equations containing our results were proposed

$$Sh \cdot ad_m = 0.97 Re_m^{0.8} N^{0.26} \quad (18)$$

for the oxygen transfer into the water,

$$Sh_1 \cdot ad_m = 1.3 \times 10^{-4} Re_m^{1.516} N^{0.74} \quad (19)$$

for the oxygen transfer into the saccharose—water solution (20 mass %), and

$$Sh_1 \cdot ad_m = 4.95 \times 10^{-9} Re_m^{1.397} N^{0.485} Sc^{1.44} \quad (20)$$

for both previous cases. Coefficient of correlation for all the cases $R \cong 0.96$.

Diffusion coefficient D_1 for the oxygen in the water was taken from Ref. [14] ($D_1 = 2.5 \times 10^{-9} \text{ m}^2 \text{ s}^{-1}$ within 18—25 °C) and that for the oxygen in the sugar solution was calculated according to the relationship given by Wilke and Chang (see Ref. [15]). The relationships (18—20) are illustrated in Figs. 8—10.

SYMBOLS

a	specific surface of mass transfer	m^{-1}
a, b	exponents	
B	constant	
C_1, C_1^*, C_e	oxygen concentration, concentration of a content saturated by the oxygen, electrode signal	%
d_m	mixer diameter	m

D	tank diameter	m
D_1	diffusivity	$\text{m}^2 \text{ s}^{-1}$
g	gravity acceleration	m s^{-2}
G	aeration parameter in eqn (16)	
j_g	air flux (superficial velocity)	$\text{m}^3 \text{ m}^{-2} \text{ s}^{-1}$
K_{1a}	volumetric overall oxygen transfer coefficient	s^{-1}
n	revolution frequency	s^{-1}
P, P_g	power, power for aerated system	W
Q	volumetric air flow rate	$\text{dm}^3 \text{ s}^{-1}$
t, t_d, t_e	time, electrode time delay, electrode time constant	s
V_l	liquid content volume	dm^3, m^3

Greek Letters

α	gas hold-up	$\text{m}^3 \text{ m}^{-3}$
ν_l	liquid kinematic viscosity	$\text{m}^2 \text{ s}^{-1}$
ρ_l	liquid density	kg m^{-3}
σ	surface tension	N m^{-1}

Nondimensional criteria:

$N = \frac{Qd_m}{\nu_l D^2}$	aeration intensity number
$Re_m = \frac{nd_m^2}{\nu_l}$	modified Reynolds number
$Sc_l = \frac{\nu_l}{D_1}$	Schmidt number for the liquid phase
$Sh_1 = \frac{k_1 d_m}{D_1}$	Sherwood number for the liquid phase

REFERENCES

1. Rychtera, M. and Páca, J., *Bioinženýrství kvasných procesů*. (Bioengineering of the Fermentation Processes.) Nakladatelství technické literatury (Publishers of Technical Literature), Prague, 1986.
2. Chriaštel, L., *Bioreaktory*. (Bioreactors.) Pp. 126, 127. Slovak University of Technology, Bratislava, 1992.
3. Kaštánek, F., Zahradník, J., Kratochvíl, J., and Čermák, J., *Chemical Reactors for Gas-Liquid Systems*. Pp. 321, 322. Academia, Prague, 1993.
4. Van't Riet, K. and Tramper, J., *Basic Bioreactor Design*. Pp. 248, 251. Dekker, New York, 1991.
5. Fryer, P. J., Pyle, D. L., and Rielly, C. D., *Chemical Engineering for the Food Industry*, p. 414. Blackie Academic, London, 1997.
6. *Biotechnology*. Vol. 2, *Fundamentals of Biochemical Engineering*. (Brauer, H., Vol. Editor.) P. 441. VCH Verlagsgesellschaft, Weinheim, 1985.
7. Aiba, S., Humphrey, A. E., and Millis, N. P., *Biochemical Engineering*. Academic Press, New York, 1973.
8. Schumpe, A., *Chem.-Ing.-Tech.* 57, 501 (1985).
9. Ľínek V., Vacek, V., Sinkule, J., and Beneš, P., *Measurement of Oxygen by Membrane Covered Probes. Guidelines for Applications in Chemical and Biochemical Engineering*. Horwood, Chichester, 1988.
10. Schuler, P., *Oxi-Fibel-Einführung in die Gelöstsauerstoff-Messtechnik*. WTW Company Manual, Weinheim, 1987.
11. Bretschneider, R. et al., *Cukrovarnické tabulky*. (Tables for the Sugar Industry.) Pp. 128, 390. Nakladatelství technické literatury (Publishers of Technical Literature), Prague, 1975.
12. Nigam, K. D. and Schumpe, A., *AIChE J.* 33, 328 (1987).
13. Mills, D. B., Bar, R., and Kirwan, D. J., *AIChE J.* 33, 1542 (1987).
14. *Chemical Engineer's Handbook*. (Perry, J. H., Editor.) Pp. 14—26. 4th Edition. McGraw-Hill, New York, 1963.
15. Coulson, J. M. and Richardson, J. F., *Chemical Engineering*, Vol. 6, p. 255. Pergamon Press, Oxford, 1983.