

Optimal Control of a Fluid-Bed Desorption Process

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To run a fluid-bed desorption process under optimum operating conditions with respect to changing independent process parameters the steady-state optimization using a mathematical model of the process was designed.

Kinetic equations of a desorption process derived from the mass and heat transfer laws depending on many parameters are not satisfactorily accurate. Rate of desorption therefore is expressed by the desorption time constant which can be determined experimentally.

The basic relationships of the controlled process are derived and an example of optimization problem is presented. Adaptive control of the process is realized by a computer which determines the optimum values of actuating quantities of the controlled process.

Heterogeneous processes frequently occur in modern chemical technology. Numerous applications in the field of catalysis give very good examples. Another group of heterogeneous processes are typical sorption processes used for separation, purification etc.

In this paper a desorption process in the fluid-bed column is studied. To design the optimum performance of this thermal process according to some technical-economic criterion, it is necessary to determine the mathematical model of the controlled process. The supervisory optimum control can be then realized by a computer which determines a new set of actuating quantity values when uncontrollable input variables are changed. This adaptive control system guarantees the optimum regime performance of the controlled process under the steady-state conditions.

In this study the system silica gel—water was investigated though the results achieved are applicable for any heterogeneous system, which satisfies general conditions put on the system investigated.

Basic relationships

Consider a fluid-bed desorption process according to Fig. 1. A narrow fraction of solid particles enters the column at a rate G (related to the pure solid) with the input amount of adsorbate w_0 in kg per kg of pure solid. The flow rate of a fluidizing gas is L (related to the pure gas) with the input adsorbate concentration h_0 . The constant hold-up in the column is Z , thermal energy input Q , fluid-bed temperature T , and surrounding temperature T_0 . The desorption takes place in the uniform fluidized layer when mixing of the solid and continuous phases can be assumed as ideal.

The average amount of adsorbate in solid particles leaving the column is w . From the steady-state material balance the output concentration h in the fluidizing gas is

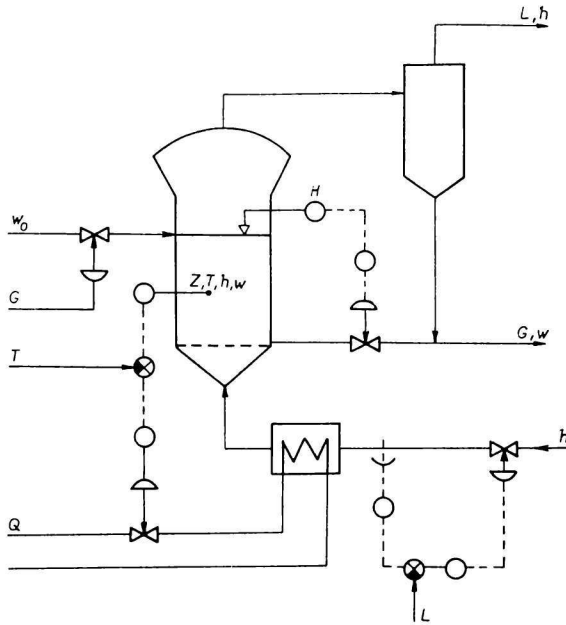


Fig. 1. Diagram of a fluid-bed desorption process.

$$h = h_0 + \frac{G}{L} (w_0 - w). \quad (1)$$

The holding time of individual particles in the column is not equal. Under the abovementioned conditions, as it is well known, the followed sample of the solid material M which entered the column in the time $t = 0$ will leave it with the flow rate [4]

$$m = \frac{M}{\tau_s} e^{-\frac{t}{\tau_s}}, \quad (2)$$

where τ_s is the system time constant

$$\tau_s = \frac{Z}{G}. \quad (3)$$

The characteristic feature of a desorption process is simultaneous heat and mass transfer. Although many authors have tried to determine analytically the coefficients of heat and mass transfer their results differ significantly.

Better results, in practice applicable, offers the experimental method of determining desorption time constant as follows:

Suppose the steady-state performance in the desorption column and the equilibrium amount of adsorbate in solid with respect to the concentration of the fluidizing gas.

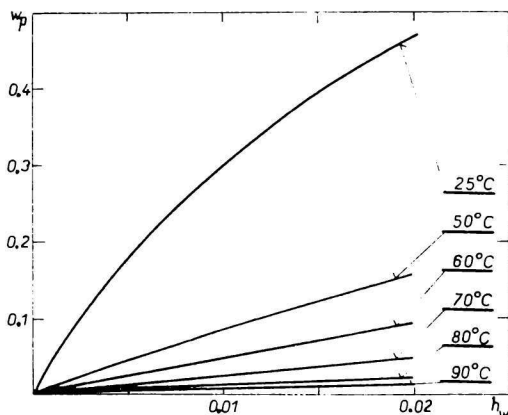


Fig. 2. Adsorption isotherms of the system silica gel—water.
 w_p and h_w are expressed in kg H_2O per kg of pure solid and gas.

This equilibrium is expressed by the adsorption isotherm. Desorption processes take place at higher temperatures when adsorption isotherms can be assumed as straight lines (cf. Fig. 2), mathematically expressed as [2]

$$h_w = Kw_p, \quad (4)$$

where K is the proportionality coefficient depending generally on temperature

$$K = f(T). \quad (5)$$

If under the steady-state conditions is put into the fluidized bed the only particle of saturated solid or some sample M small enough not to change the steady-state operating conditions we can write the desorption rate equation [1]

$$(h_w - h)K_p F_p = - \frac{dw_p}{dt}, \quad (6)$$

where w_p is the amount of adsorbate in the solid particle, h_w is the equilibrium concentration, K_p is the over-all mass transfer coefficient, and F_p is the surface of the particle. With respect to (4) and denoting

$$\tau_r = \frac{1}{KK_p F_p} \quad (7)$$

equation (6) can be written in the following way

$$\tau_r \frac{dw_p}{dt} + w_p = \frac{h}{K}. \quad (8)$$

Assume the initial amount of adsorbate in the particle is w_0 , the desorption process in time is

$$w_p = \frac{h}{K} + \left(w_0 - \frac{h}{K} \right) e^{-\frac{t}{\tau_r}}. \quad (9)$$

The desorption time constant τ_r is the characteristic quantity of the desorption process kinetics and can be determined experimentally:

A small sample M of saturated solid (ca. 4–8% of the hold-up Z) is put into the column under the steady-state conditions and then the amount of adsorbate in solid is measured by an automatic analyzer or by analyzing samples taken off in certain time intervals. From the experimental curve which is corresponding with the equation (9) the desorption time constant τ_r can easily be determined graphically (cf. Fig. 3).

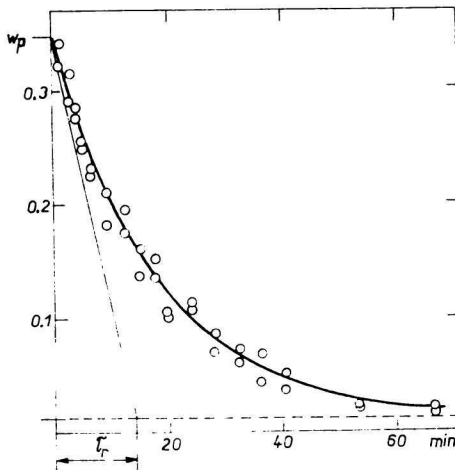


Fig. 3. Desorption process in time. w_p is expressed in kg H₂O per kg of pure solid.

This experiment is to be made under various operating conditions, changing temperature T at the stable flow rate L and *vice versa*. The general relationship

$$\tau_r = f(T, L) \quad (10)$$

in the investigated system is well approximated by the two-dimensional equation

$$\tau_r = k_1 + k_2 T + k_3 T^2 + (k_4 + k_6 T)L, \quad (11)$$

where k_1, k_2, \dots, k_5 are constants.

It can be easily derived that the average output amount of adsorbate in solid is [1]

$$w = \frac{1}{M} \int_0^{\infty} m(t) w_p(t) dt. \quad (12)$$

Substituting (2) and (9) in (12) and solving the integral (12) we get

$$\tau_s = \tau_r \left(\frac{w_0 - w}{w - \frac{h}{K}} \right), \quad (13)$$

hence

$$w = \frac{h}{K} + \left(w_0 - \frac{h}{K} \right) \frac{\tau_r}{\tau_r + \frac{Z}{G}} \quad (14)$$

From the heat energy balance under steady-state conditions follows

$$Q = G(w_0 - w)r + (Gc_{ps} + Gw_0c_{pv} + Lc_{pa} + Lh_0c_{pp})(T - T_0) + Q_w, \quad (15)$$

where Q is the input heat energy, c_{ps} , c_{pa} , c_{pv} , c_{pp} are the heat capacities of pure solid, pure fluidizing gas, liquid and gaseous adsorbate. The heat of adsorption is denoted as r and Q_w is the lost thermal energy approximated by equation

$$Q_w = (b_1 + b_2L)(T - T_0), \quad (16)$$

where b_1 , b_2 are constants and T_0 is surrounding temperature. Substituting (16) in (15) we get

$$T = T_0 + \frac{Q - G(w_0 - w)r}{Gc_{ps} + Gw_0c_{pv} + Lc_{pa} + Lh_0c_{pp} + b_1 + b_2L} \quad (17)$$

Mathematical model of the process

The mathematical model of a process is a completed set of equations which expresses the relationships among the input and output quantities. The block diagram of the investigated desorption process is on Fig. 4.

Purposeful changes in the process regime can be realized by changing actuating quantities L , G , Q , while the independent input quantities T_0 , w_0 , h_0 , are supposed as disturbance variables.

Relationships (I), (5), (11), (14), (17), form the mathematical model of the investigated fluid-bed desorption process. These equations can be programmed either for analog or digital computer. The circuit diagram for the analog computer is on Fig. 5.

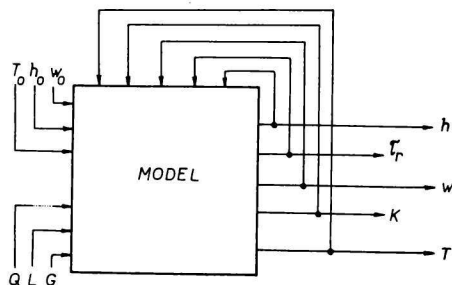


Fig. 4. Block diagram of a fluid-bed desorption process.

By a digital computer were calculated the static characteristics of the investigated fluid-bed desorption process at constant values of disturbance quantities T_0 , w_0 , h_0 , as well as two of three actuating quantities (cf. Fig. 6–8).

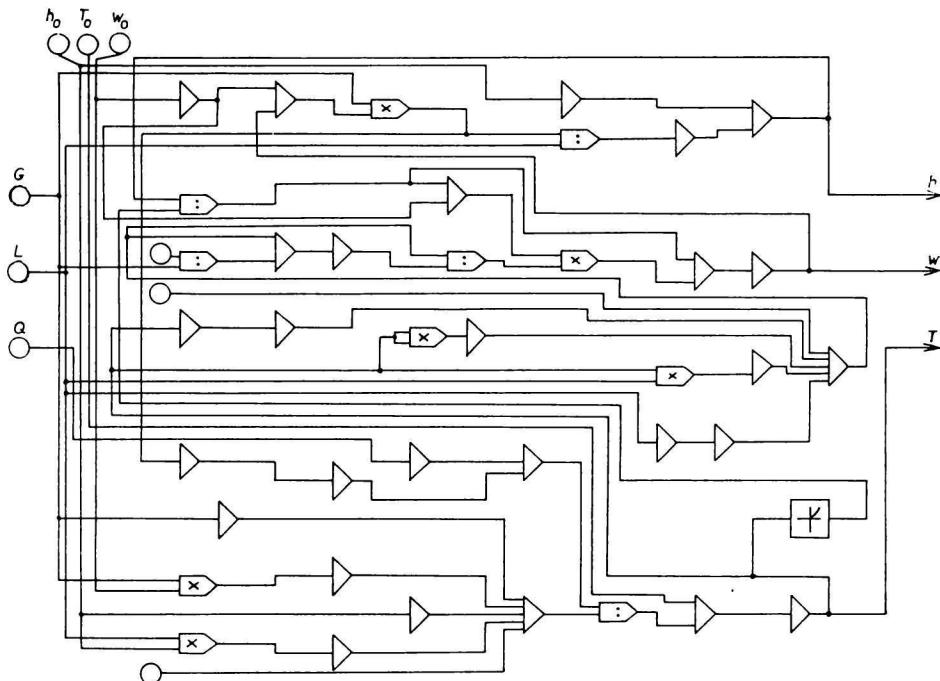


Fig. 5. Circuit diagram of the fluid-bed desorption process.

Optimization problem

In a very simple way expressed formulation of optimum process performance is to maximize economic return or minimize production cost under given operating conditions and process limitations. Mathematical formulation of the optimum performance criterion is called the objective function, generally depending on input and output quantities as well as on economic quantities which represent material and energy cost, stable production cost etc. The optimum process performance under given operating conditions can be reached by choosing proper values of the actuating variables. The calculations of the optimum values are usually computed on analog or digital computer, once and again, when the independent process parameters were changed.

Suppose that in the investigated fluid-bed desorption process the only economically valuable product is the solid material. Denoting

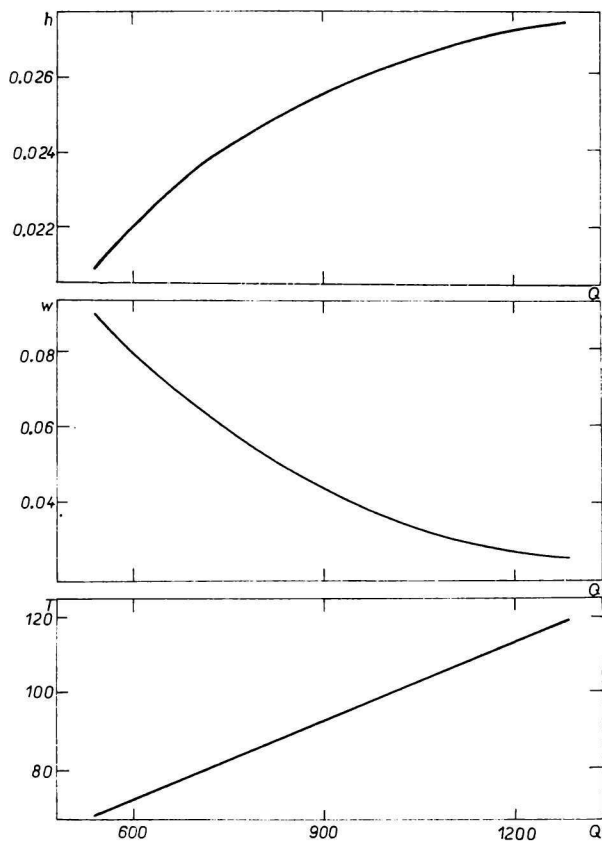


Fig. 6. Static characteristics — independent variable is the input thermal energy Q in kcal/hour.

- c_1 — the output cost of the solid material
- c_2 — the input cost of the solid material
- c_3 — the unit cost of thermal energy
- c_4 — the unit cost of the fluidizing compressed gas
- c_5 — the stable production cost

the objective function is

$$E = G(c_1 - c_2) - (Gc_{ps} + Gc_{pv}w_0 + Lc_{pa} + Lc_{pp}h_0 + b_1 + b_2L)(T - T_0)c_3 - G(w_0 - w_x)rc_3 - Lc_4 - c_5. \quad (18)$$

To produce the solid material of desired quality w_x , according to equation (13), the following limiting condition must be satisfied

$$(k_1 + k_2T + k_3T^2 + k_4L + k_5TL) \left(\frac{w_0 - w_x}{w_x - \frac{h}{K}} \right) - \frac{Z}{G} = 0. \quad (19)$$

Other process variable limitations are

$$\begin{aligned} G_{\min} &\leq G \leq G_{\max}, \\ L_{\min} &\leq L \leq L_{\max}, \\ T_{\min} &\leq T \leq T_{\max}. \end{aligned} \quad (20)$$

Then the Lagrange's function is [3]

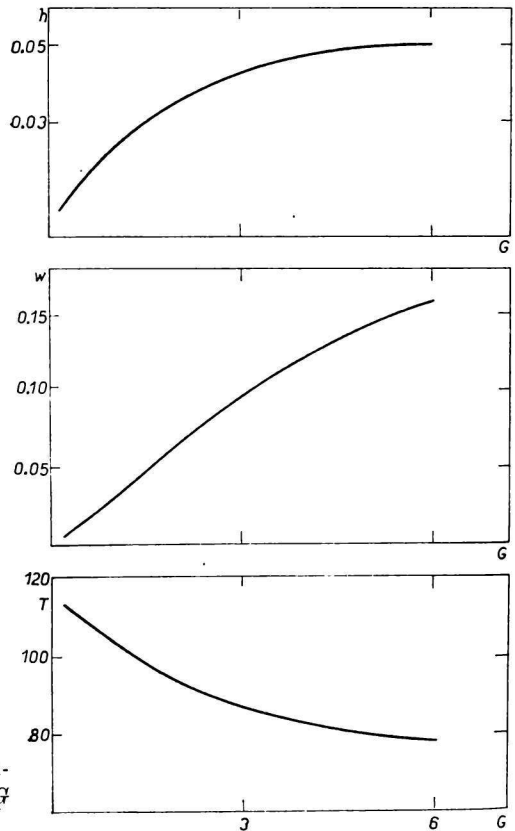


Fig. 7. Static characteristics — independent variable is the mass flow rate G in kg/hour.

$$\Phi = G(c_1 - c_2) - (Gc_{ps} + Gc_{pv}w_0 + Lc_{pa} + Lc_{pp}h_0 + b_1 + b_2L)(T - T_0)c_3 - G(w_0 - w_x)rc_3 - Lc_4 - c_5 - \lambda \left[(k_1 + k_2T + k_3T^2 + k_4L + k_5TL) \left(\frac{w_0 - w_x}{w_x - \frac{h}{K}} \right) - \frac{Z}{G} \right], \quad (21)$$

where λ is the Lagrange's multiplier. The partial derivatives of the Lagrange's function with respect to G , L , T , λ , take up the form

$$\frac{\partial \Phi}{\partial G} = c_1 - c_2 - (c_{ps} + c_{pv}w_0)(T - T_0)c_3 - (w_0 - w_x)rc_3 - \lambda \frac{Z}{G^2},$$

$$\frac{\partial \Phi}{\partial L} = -(c_{pa} + c_{pp}h_0 + b_2)(T - T_0)c_3 - c_4 + \lambda(k_4 + k_5T) \left(\frac{w_0 - w_x}{w_x - \frac{h}{K}} \right),$$

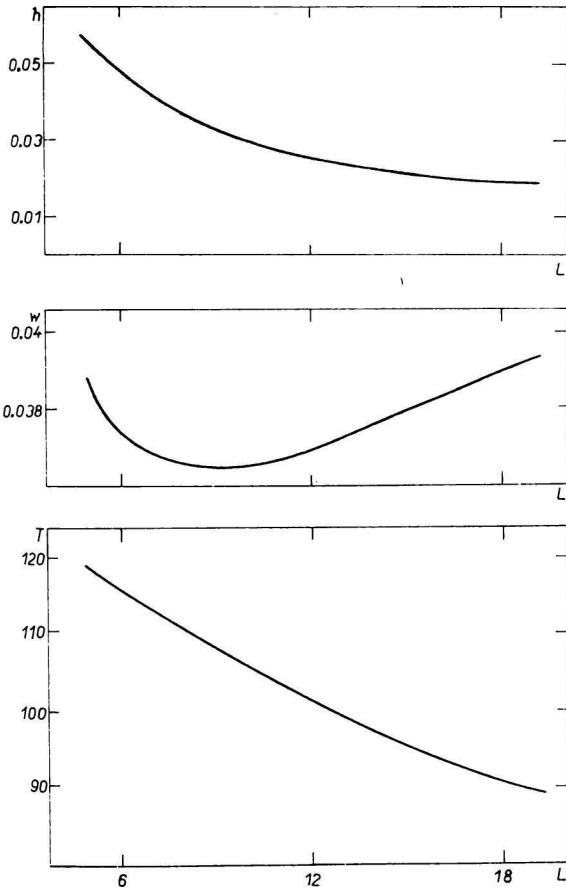


Fig. 8. Static characteristics — independent variable is the flow rate L in kg/hour.

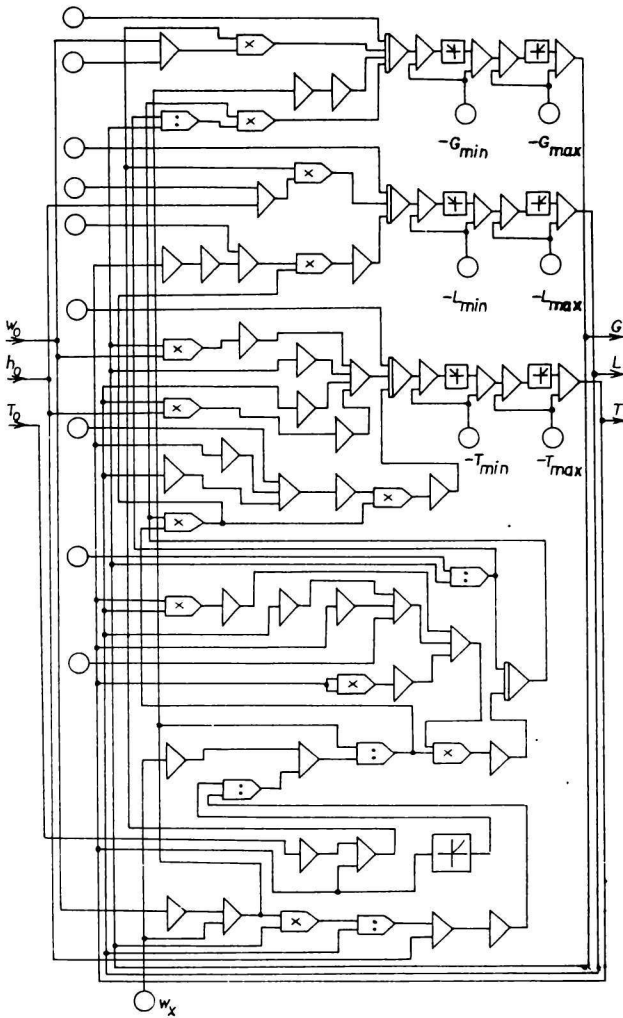


Fig. 9. Circuit diagram of the optimizer.

$$\frac{\partial \Phi}{\partial T} = -(Gc_{ps} + Gc_{pv}w_0 + Lc_{pa} + Lc_{pp}h_0 + b_1 + b_2L)c_3 +$$

$$+ \lambda(k_2 + 2k_3T + k_5L) \left(\frac{w_0 - w_x}{w_x - \frac{h}{K}} \right), \quad (22)$$

$$\frac{\partial \Phi}{\partial \lambda} = - \left[(k_1 + k_2T + k_3T^2 + k_4L + k_5TL) \left(\frac{w_0 - w_x}{w_x - \frac{h}{K}} \right) - \frac{Z}{G} \right].$$

Common techniques for solving systems of non linear equations are the gradient methods. Solutions are performed usually on analog or digital computer.

The circuit diagram of the analog optimizer according to equations (22) with respect to limitations (20) is on Fig. 9. The input variables of the optimizer are the independent process quantities T_0 , w_0 , h_0 , while the output variables of the optimizer are the optimum values of the mass feed rate G , the flow rate of fluidizing gas L and the fluid-bed temperature T .

With respect to the character of the fluid-bed desorption process and technological limitations, the steady-state optimization is proposed. For characterization of the process kinetics the desorption time constant was introduced and the basic relationships were derived to design the mathematical model of the process. This study was intended as a methodological approach to design the optimum steady-state conditions of the fluid-bed desorption process.

Symbols

b_1, b_2	Constants in equations (16), (17)
$c_1 \dots c_5$	Constants in equation (18)
e	Base of Napierian logarithms
f	General functionship
h	Output concentration in the fluidizing gas
h_0	Input concentration in the fluidizing gas
h_w	Equilibrium concentration in equations (4), (6)
$k_1 \dots k_5$	Constants in equation (11)
m	Mass flow rate of the followed sample M
r	Heat of adsorption
t	Time
w	Average amount of adsorbate in output solid
w_0	Average amount of adsorbate in input solid
w_p	Amount of adsorbate in solid in equations (4), (6), (8), (9)
w_r	Desired amount of adsorbate in output solid
E	Objective function
F_p	Surface of the particle in equation (6), (7)
G	Mass feed rate of solid material
K	Coefficient in linear adsorption isotherm
K_p	Mass transfer coefficient in equation (6), (7)
L	Flow rate of the fluidizing gas
M	Mass of the followed sample
T	Fluid-bed temperature
T_0	Surrounding temperature
Q	Input thermal energy
Q_w	Lost thermal energy
Z	Hold-up of solid material
λ	Lagrange's multiplier
τ_s	Time constant
τ_r	Desorption time constant
Φ	Lagrange's function

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