# Suspended Particle Circulation in Gas-Lift Tanks 

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#### Abstract

Introduction of gas to liquid in a vertical tube results in a hold-up of bubbles and related decrease of average density. It can be employed in air-lift pumps for transporting liquid upwards or in a closed loop to agitate liquid in gas-lift reactors. Popular equipment of hydrometallurgy is a pachuca tank, large slender cylinder with an air-lift riser in central position. It is used for leaching processes, where abrasive particles are handled in large volumes of extremely corrosive liquids, and therefore moving parts should be avoided. Amount of gas, required to agitate liquid efficiently and to keep solid particles suspended is an important quantity. Suggested model takes into consideration rising velocity of bubbles, settling velocity of particles, and pressure loss. Friction coefficient is determined from pilot plant data with air-water system. Superficial velocity of liquid as a function of gas flow rate for pure liquids and superficial velocity of solid particles as a function of gas flow rate for the settling suspensions are results of the simulation of the process. Power input and its fraction employed to keeping particles suspended, i.e. efficiency of the suspension is discussed.


Hold-up of bubbles in a particular volume of liquid results in decreasing average density and consequently in the upward flow of the liquid with respect to gas-free liquid. Introduction of gas to the bottom of a submerged tube moves liquid up, which is employed in air-lift pumps. The air-lift pump requires large volume flow of pressurized gas and discharge of liquid is comparatively small at higher liquid heads. Efficient mammoth air-lift pumps for transporting huge amount of liquid from large depth belong to the science fiction. However, the air-lift is applicable in laboratory continuous models handling liquids [1] where its simplicity, absence of moving parts and sealing, and possibility to use various gases for reactive, corrosive or flammable liquids is beneficial. In industrial scale, there is adverse fact that compression of gas requires more expensive equipment (turbocompressor) than conventional pumps handling liquids. Other situation is when low head and high discharge is demanded as in gas-liquid reactors. In particular during the aerobic fermentation in bioreactors, gas can be employed to the liquid agitation supplied if introduced to the riser tube, which serves as an air-lift.

Other popular industrial application is in leaching vessels of hydrometallurgy processes, where abrasive particles in extremely corrosive liquids should be handled in large volumes, and therefore moving parts should be avoided. Here, air-lift represents an infallible technique of mixing which can also suspend settled particles, e.g. after a temporary interruption of the process [2]. In this application, the tanks with air-lift
used to be arranged in the geometry called pachuca tanks (silver mines at the Mexican town Pachuca was the place where this equipment became common). It is a cylindrical tank with a conical bottom to prevent settling particles in corners. From the viewpoint of hydrodynamics, diameter $d$ of the air-lift tube, and height of the bubble column $H_{0}$ is essential. Diameter $D$ is usually considerably larger than $d$. In sixties and seventies, several tens of pachuca tanks with nominal volume $300 \mathrm{~m}^{3}$ with $d=0.8 \mathrm{~m}, D=4.5 \mathrm{~m}, H_{0}=18 \mathrm{~m}$ were erected in the Czechoslovak industry of uranium (Fig. 1).

Military character of the uranium industry was a reason for the performance of the technology deeply on the side of infallibility even for the price of endless wasting of energy and serious damage of environment. Recent engineering analysis and mathematical modeling shows that there is a space for considerable savings and the equipment is still suitable for leaching of ores and other processes in hydrometallurgy. The mathematical model seems to be quite simple. Nevertheless in this scale, number of effects that can be omitted in a pilot plant research, stays to be important. This paper presents a model based on a knowledge of bubble rise velocity, hydraulic resistance of a circulatory flow in a pachuca tank, and terminal settling velocity, to predict the motion of a suspension of settling particles and their concentration as a function of the gas flow rate.

The fact that aerobic fermentors with air-lift have been extensively studied yet and there is a number of


Fig. 1. Laboratory model 12 of the pachuca (dimensions/mm).
papers on the subject is not of very much use for our purpose. Sedimentation is not on issue in bioreactors and gas is supplied with the aim to provide oxygen and its air-lift effect is more or less unwitting. The pachucas have also quite different geometry than common aerobic fermentors. In literature, we can find a number of papers dealing with hydrodynamics of air-lift tanks [2-5]. Most of them present simple correlation of experimental gas and liquid volume flow rates, gas hold-up, and sometimes mass transfer for particular pilot plant equipment. Several authors studied liquid flow pattern outside the riser [6-8], however no data on pressure distribution and friction losses are available.

The bubble rise velocity, $v_{\mathrm{B}}$, friction coefficient, $\zeta$, of the circulatory flow, and terminal settling velocity, $v_{\mathrm{P}}$, of the particles are key quantities required in the mathematical model of the process. It is important that in low-viscosity liquids, the bubbles in a swarm irrespectively to their origin, assume fast a terminal size and terminal velocity [9] which is usually $v_{\mathrm{B}}=$ $0.18 \mathrm{~m} \mathrm{~s}^{-1}$.

## THEORETICAL

## Friction Coefficient

The above data can be employed to determine the friction coefficient if a simplified model of the process is set up.

Volume fraction of gas and liquid in the riser is $\varphi_{\mathrm{G}}$ and ( $1-\varphi_{G}$ ), respectively. Then, actual velocity of liquid is

$$
\begin{equation*}
v_{\mathrm{L} 1}=u_{\mathrm{L}} /\left(1-\varphi_{\mathrm{G}}\right) \tag{1}
\end{equation*}
$$

We can assume that the rising velocity of bubbles with respect to liquid is $v_{\mathrm{B}}$, and their actual velocity with respect to the equipment is $\left(v_{\mathrm{L} 1}+v_{\mathrm{B}}\right)$. It means that

$$
\begin{equation*}
\left(v_{\mathrm{L} 1}+v_{\mathrm{B}}\right) \varphi_{\mathrm{G}}=u_{\mathrm{G}} \tag{2}
\end{equation*}
$$

At the level the pressure is $p_{0}$ and when the air-lift length is $H$, hydrostatic pressure at its bottom is

$$
\begin{align*}
p_{1}= & p_{0}+\left[\rho_{\mathrm{L}}\left(1-\varphi_{\mathrm{G}}\right)+\rho_{\mathrm{G}} \varphi_{\mathrm{G}}\right] g H \approx \\
& \approx p_{0}+\rho_{\mathrm{L}}\left(1-\varphi_{\mathrm{G}}\right) g H \tag{3}
\end{align*}
$$

The hydrostatic pressure of the same outer space is

$$
\begin{equation*}
p_{2}=p_{0}+\rho_{\mathrm{L}} g H \tag{4}
\end{equation*}
$$

and there is a pressure difference

$$
\begin{equation*}
\Delta p=\rho_{\mathrm{L}} \varphi_{\mathrm{G}} g H \tag{5}
\end{equation*}
$$

which is consumed by the friction loss. The friction factor $\zeta$ is defined as a ratio of actual pressure loss to the kinetic energy of unit volume in the air-lift

$$
\begin{equation*}
\Delta p=\zeta\left(\rho_{\mathrm{L}} / 2\right) v_{\mathrm{L} 1}^{2}\left(1-\varphi_{\mathrm{G}}\right) \tag{6}
\end{equation*}
$$

In low-viscosity liquids at higher Reynolds numbers under consideration, we can assume that $\zeta$ is a constant for a class of geometrically similar flow situations. In the classical paper Lamont [11] estimated $\zeta$ $=2.8$ by a simple addition of the hydraulic resistance of a pipe including entrance and end losses. Here, we will determine actual friction coefficient from the experimental data by Svozil and Seichter [10].

Solution of eqns (1),(2),(5),(6), after introduction of dimensionless variables

$$
\begin{gather*}
u_{\mathrm{G}}^{*} \equiv u_{\mathrm{G}} / \sqrt{(2 g H / \zeta)}  \tag{7}\\
u_{\mathrm{L}}^{*} \equiv u_{\mathrm{L}} / \sqrt{(2 g H / \zeta)}  \tag{8}\\
B \equiv \zeta v_{\mathrm{B}}^{2} /(2 g H) \tag{9}
\end{gather*}
$$

can be expressed by the implicit irrational function


Fig. 2. Dimensionless superficial velocity of liquid as a function of gas superficial velocity for particular values of the parameter B. -.-- $B=0.1 ;-B=0.01 ; \cdots B=0.001 ;---B=0.0001$.


Fig. 3. Dimensionless superficial velocity of liquid as a function of gas superficial velocity for particular values $H$. The values are normalized employing $\zeta=4.5$. $\quad H=1525 \mathrm{~mm} ; H=1471 \mathrm{~mm} ; \boldsymbol{\Delta} H=1417 \mathrm{~mm} ; \quad \bullet=1307 \mathrm{~mm} ;+H=1180 \mathrm{~mm}$; $-B=0.005$.

$$
f\left(u_{\mathrm{L}}^{*}, u_{\mathrm{G}}^{*}, B\right)=0
$$

We have investigated properties of this function by numerical experiments. Dependence of $u_{\mathrm{L}}^{*}$ on $u_{\mathrm{G}}^{*}$ for a large set of parameter $B$ is presented in Fig. 2. Apparently, it is not essentially sensitive to $B$. It was employed to determine the friction coefficient, $\zeta$.

## EXPERIMENTAL

## Liquid Circulation

Experimental study of the hydrodynamics of a pachuca tank has been performed by Svozil and Seichter [10] (Fig. 1). The Perspex bench scale tank of 390 mm diameter and 1298 mm length was manufactured with an air-lift tube of $\varnothing 70 \mathrm{~mm}$ (model $1 \quad 12$ of the industrial tank). During a set of tests with different height of level, the effect of air flow rate on
the water circulation was studied. The water circulation time was determined from the interval of peaks in electric conductivity response to the addition of a tracer, which was the solution of electrolyte. Gas was introduced alternately by the tube of $\varnothing 7.5 \mathrm{~mm}$ or by a sieve sparger of $\varnothing 30 \mathrm{~mm}$. Superficial velocities of gas and liquid in the riser, $u_{G}, u_{L}$, have been calculated.

Numerical experiments during the search of optimum value $\zeta$ are easily performed in a spreadsheet, where the fit to a given plot $u_{\mathrm{L}}^{*}\left(u_{\mathrm{G}}^{*}, B\right)$ is immediately available. It was found that a group of experiments with liquid level above the upper edge of riser tube is fitted by a single line $u_{\mathrm{L}}^{*}\left(u_{\mathrm{G}}^{*}\right)$, and the best fit in the linear coordinates is with

$$
\begin{equation*}
\zeta=4.5 \tag{10}
\end{equation*}
$$

The fit is apparent in Fig. 3.
In the set of data with lower liquid level, some ad-
ditional friction loss should be considered. However, this is not the case of interest here.

The experimental value $\zeta=4.5$ is considerably higher than the estimate by Lamont [11]. He considered only the entrance and end effects of the flow in a system large vessel-pipe-large vessel. Complete change of the flow direction was not taken into account. Therefore, it is not surprising that his value 2.8 was underestimating.

## RESULTS AND DISCUSSION

## Suspension of Coarse Particles in Pachuca Tanks

Knowledge of $B$ and $\zeta$ for the air-lift can be also employed for the prediction of flow of suspensions. Further quantity needed in the analysis is the terminal settling velocity, $v_{\mathrm{P}}$, of the particles.

When the suspension contains spherical particles of density $\rho_{\mathrm{S}}$ and diameter $d_{\mathrm{P}}$, the terminal settling velocity, $v_{\mathrm{P}}$, can be theoretically predicted. In this case we are characterizing the particles just by $v_{\mathrm{P}}$ without respect to their size and shape. While bubbles upward velocity is the liquid velocity plus $v_{\mathrm{B}}$, particle upward velocity is the liquid velocity minus $v_{\mathrm{p}}$.

In the air-lift, there is the upward liquid velocity $v_{\mathrm{L} 1}$, particle velocity $v_{\mathrm{L} 1}-v_{\mathrm{P}}$, and bubble velocity $v_{\mathrm{L} 1}$ $+v_{\mathrm{B}}$. The hold-up of gas and gas velocity is related to the gas superficial velocity

$$
\begin{equation*}
\left(v_{\mathrm{L} 1}+v_{\mathrm{B}}\right) \varphi_{\mathrm{G}}=u_{\mathrm{G}} \tag{11}
\end{equation*}
$$

For solid it is

$$
\begin{equation*}
\left(v_{\mathrm{L} 1}-v_{\mathrm{P}}\right) \varphi_{\mathrm{S} 1}=u_{\mathrm{S} 1} \tag{12}
\end{equation*}
$$

and for liquid

$$
\begin{equation*}
v_{\mathrm{L} 1}\left(1-\varphi_{\mathrm{G}}-\varphi_{\mathrm{S} 1}\right)=u_{\mathrm{L}} \tag{13}
\end{equation*}
$$

Evidently, if particles are suspended, $v_{\mathrm{L} 1}-v_{\mathrm{P}}$ should be positive.

The cases where the ratio of solids and liquid volumes

$$
\begin{equation*}
X \equiv \varphi_{\mathrm{S} 1} /\left(1-\varphi_{\mathrm{S} 1}-\varphi_{\mathrm{G}}\right) \tag{14}
\end{equation*}
$$

is moderate, say $X \ll 0.2$, are considered. The viscosity of suspensions beyond this limit significantly rises and sooner or later the mixture loses its liquid-like behaviour.

Downward flow has liquid velocity $v_{\mathrm{L} 2}=\left(A_{1} / A_{2}\right) v_{\mathrm{L} 1}$ and solid velocity is ( $v_{\mathrm{L} 2}+v_{\mathrm{P}}$ ). In vessels with comparatively small area occupied by the riser $A_{1} \ll A_{2}$ we have approximately $v_{\mathrm{L} 2}+v_{\mathrm{P}} \approx v_{\mathrm{P}}$ and residence time of solids in the outer space of vessel is $H / v_{\mathrm{p}}$. However, the concentration of solids $\varphi_{\mathrm{S} 2} \ll \varphi_{\mathrm{S} 1}$ here
may be low and the average density of liquid is close to $\rho_{\mathrm{L}}$.

Driving force for the circulatory flow is given by the difference of hydrostatic pressure outside and inside of the riser

$$
\begin{equation*}
\Delta p=\left[\rho_{\mathrm{L}}\left(\varphi_{\mathrm{G}}+\varphi_{\mathrm{S} 1}\right)-\rho_{\mathrm{S}} \varphi_{\mathrm{S} 1}\right] g H \tag{15}
\end{equation*}
$$

This force is consumed by friction. We suppose the friction to be proportional to the kinetic energy of liquid and solid upward flow (minor gas kinetic energy can be neglected). Then,

$$
\begin{align*}
\Delta p= & \zeta\left[\left(\rho_{\mathrm{L}} / 2\right) v_{\mathrm{L}_{1}}^{2}\left(1-\varphi_{\mathrm{G}}-\varphi_{\mathrm{S} 1}\right)+\right. \\
& \left.+\left(\rho_{\mathrm{S}} / 2\right)\left(v_{\mathrm{L} 1}-v_{\mathrm{P}}\right)^{2} \varphi_{\mathrm{S} 1}\right] \tag{16}
\end{align*}
$$

Velocities in the system assume just the values satisfying relations (11-16).

Dimensionless transformation uses beside of eqns (7-9) also

$$
\begin{gather*}
s \equiv \rho_{\mathrm{S}}^{*} / \rho_{\mathrm{L}}  \tag{17}\\
u_{\mathrm{P}}^{*} \equiv v_{\mathrm{P}} / \sqrt{(2 g H / \zeta)}  \tag{18}\\
u_{\mathrm{S} 1}^{*} \equiv u_{\mathrm{S} 1} / \sqrt{(2 g H / \zeta)} \tag{19}
\end{gather*}
$$

According to the model, the functions $u_{\mathrm{L}}^{*}\left(u_{\mathrm{G}}^{*}\right)$ and $u_{\mathrm{S}}^{*}\left(u_{\mathrm{G}}^{*}\right)$ depend on 3 other parameters or their combinations, e.g. $X, s$, and $B$ remain. The material parameter $s$ is for leaching of ores usually equal to 2.75 . The parameter $B$ characterizes height of the air-lift, in industrial scale $B=0.0004$, the pilot plant experiments were carried out with $B=0.005$. The last parameter $X$ characterizes a concentration of suspension in the riser and then also limits of the applicability of air-lift mixing.

Results of numerical experiments are interpreted by the functions: $u_{\mathrm{L}}^{*}\left(u_{\mathrm{G}}^{*}, X, s, B\right)$ and $u_{\mathrm{S} 1}^{*}\left(u_{\mathrm{G}}^{*}, X, s, B\right)$. In Fig. 4 numerical parameters $X, s$, and $B$ are presented as a triplet of numbers in the legend.

## Superficial Velocities

At very low gas velocities, suspension of solids fails.
In any case there is a maximum of both liquid and solid flow rate at increasing gas flow rate. Gas occupies nearly all volume of the riser and there is a little space left for the suspension.

The pumping of suspensions is efficient at lower $u_{\mathrm{G}}^{*}<1$. We can understand why a common correlation of experimental data for air-lifts is often presented as a proportionality $u_{\mathrm{L} 1} \approx u_{\mathrm{G}}^{0.4}$.

It is interesting that there is some cover line of the functions $u_{\mathrm{S} 1}^{*}\left(u_{\mathrm{G}}^{*}\right)$. It means that decreasing liquid velocity compensates increasing solid concentration in air-lifts.


Fig. 4. Dimensionless superficial velocities of liquid and solid as a function of gas flow rate for $s=2.75$. Three plots are for different $B$, a) $B=0.01$, b) $B=0.001$, c) $B=$ 0.0001 . Upper group of curves for different $X$ belongs to $u_{\mathrm{L}}^{*}$, lower group belongs to $u_{\mathrm{S}}^{*}$. $-\cdots-X=0.02$; ---$X=0.05 ; \cdots-X=0.10 ;-X=0.20$.

## Energy for Solid Suspension

In the air-lift reactors with homogeneous liquids or with slowly settling suspensions, the energy is con-
sumed to overcome hydraulic resistance. Here, the energy is supplied to keep particles suspended. The power loosed by sedimentation is

$$
\begin{equation*}
P_{\text {sus }}=v_{\mathrm{P}}\left(\rho_{\mathrm{S}}-\rho_{\mathrm{L}}\right) g A_{1} H\left(\varphi_{\mathrm{S} 1}+u_{\mathrm{S} 1} / v_{\mathrm{P}}\right) \tag{20}
\end{equation*}
$$

Power input depends on gas volume flow, $A_{1} u_{G}$, and pressure. In simple case it is just the product

$$
\begin{equation*}
P_{\mathrm{in}}=A_{1} u_{\mathrm{G}} t \rho_{\mathrm{L}} g H \tag{21}
\end{equation*}
$$

The fraction employed to keep particles suspended is called efficiency

$$
\eta=P_{\text {sus }} / P_{\text {in }}
$$

The efficiency is nearly 1 (or $\mathbf{1 0 0 \%}$ ) for heavy settling suspensions at lower velocities $u_{\mathrm{G} 0}^{*}<0.1$.

The power needed to keep unit mass of solids suspended is

$$
\begin{align*}
\Phi_{\text {sus }} & =P_{\text {sus }} /\left[A_{1} H\left(\varphi_{\mathrm{S} 1}+u_{\mathrm{S} 1} / v_{\mathrm{P}}\right)\right]= \\
& =v_{\mathrm{P}} g\left(\rho_{\mathrm{S}}-\rho_{\mathrm{L}}\right) / \rho_{\mathrm{S}} \tag{22}
\end{align*}
$$

It is comparatively large and makes about $\Phi_{\text {sus }}=0.4$ $\mathrm{W} \mathrm{kg}^{-1}$ for leaching of common grinded ores and values like $P_{\text {sus }}=50-100 \mathrm{~kW}$ may be on issue in typical industrial pachucas. Required gas superficial velocity in the riser is $u_{G}=(0.4 / \eta) \mathrm{m} \mathrm{s}^{-1}$.

## Extended Problems

Presented simplified model demonstrates a possibility how to solve some problems of solid particle suspension in pachuca tanks. The industrial processes are more sophisticated, and number of other variables have to be taken into consideration, however similar approach to the solution of flow and concentration of particular phases could be applied. Some examples of additional problems are listed:

- Sometimes, finite values ( $A_{1} / A_{2}$ ) must not be neglected and the ratio enters as a further parameter into the mathematical model.
- A single fraction of settling particles was considered here. Usually, there is some distribution of settling velocities, and in certain process other kind of particles (e.g. ion-exchanging resin) is present. Generally, presence of fine fractions balances the densities in a riser and in the vessel and improves circulation. In continuous processes, various particles have different residence time, which affects overall mass transfer.
- The model can be solved for unsteady-state conditions. Comparatively long response to any input parameter is important as the reactors are usually arranged in cascades.
- Constant gas superficial velocity has been assumed in the model. Different situation is in industrial pachuca tanks where density of gas depends signifi-
cantly on the hydrostatic pressure. Similar problems are in steelworks where high-density molten metal is bubbled under a low pressure at the level.
- In gas-lift reactors with heat and mass transfer, there may occur not only volume changes, but also mass flow rate changes in gas flow. In the tanks with hot liquid, saturation of air by steam occurs. In some processes gas may also enter a reaction or it is produced.

Now, such multiparameter problems are solved as case studies. Whenever any new general conclusions appear, they will be published in the following papers.

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## SYMBOLS

$A_{1} \quad$ cross-section of the riser
$A_{2}$ cross-section of the outer space in a tank
$B$ dimensionless parameter, eqn (9)
d diameter of the riser
$D$ diameter of the vessel
$H$ height of the air-lift
$P_{\text {in }}$ power input
$P_{\text {sus }}$ power for solid suspension
$s \quad$ density ratio, eqn (17)
$u_{\mathrm{G}} \quad$ superficial velocity of gas in riser
$u_{\mathrm{L}} \quad$ superficial velocity of pure liquid in riser
$u_{\mathrm{S}} \quad$ superficial velocity of solids in riser
$v_{\mathrm{B}} \quad$ rising velocity of bubbles in a swarm
$v_{\mathrm{L} 1}$ actual velocity of pure liquid in riser
$v_{\text {L2 }}$ actual downward velocity of pure liquid in tank
$v_{\mathrm{P}} \quad$ terminal settling velocity of particles
$X \quad$ volume ratio of solids to pure liquid
$\zeta$ friction coefficient
$\rho_{\mathrm{L}}$ density of liquid
$\rho_{\mathrm{S}} \quad$ density of solids
$\varphi_{\mathrm{G}} \quad$ volume fraction (hold-up) of gas in riser
$\varphi_{\mathrm{S} 1} \quad$ volume fraction (hold-up) of solids in riser dimensionless form

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