Adaptive Control of a Laboratory Tank System

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A computer-based control of a laboratory tank system is described. Model Reference Adaptive Control (MRAC) is one of many approaches to adaptive control. The advantage is that the MRAC is the adaptive control without an identification "on-line". The behaviour of unknown plant is prescribed by the behaviour of the reference model. MRAC is used to control a first-order system and higher-order systems, respectively.

In a lot of industrial branches the storage tank system is used very often. In chemical industry, for example, the use of the storage tanks is a necessity. The level of each tank is necessary to watch at each moment of the time because of the technology and safety of the process. From this point of view the level control of the storage tank system is always topical [1]. Laboratory model consists of three tanks, N_1, N_2, and N_3. They can be connected and influenced to each other or not. The valves V_1, V_2, and V_3 are on the outputs of each tank and they enable to set the output flow rate. The goal of the control of hydraulic system is to achieve the control value of the level in the tank N_3 by changing the feed flow q_0. The valves have nonlinear flow responses. The level h_3 in tank N_3 is measured by the capacitive level sensor and its output signal is fed into the control computer through A/D converter. The output signal from computer controls the inlet flow q_0 to the tank. More details about the technical realization and methods to set time constants of experimental model were published in [2]. The laboratory tank system can be seen in Fig. 1.

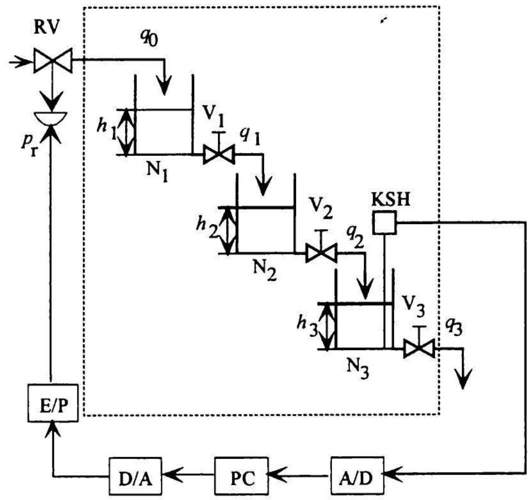

Fig. 1. Control structure of the laboratory tank model. KSH – the capacitive level sensor, PC – the personal computer, RV – the control valve; E/P, A/D, and D/A – signal converters.

THEORETICAL

Fig. 1 shows a case when the tanks are connected without interaction. The relationships between input and output variables of the tank N_i are described by the steady-state equation [2]

$$F_i \frac{dh_i}{dt} = q_{i-1} - q_i \tag{1}$$

where F_i (m^2) is the tank area, q_i ($m^3\ s^{-1}$) the flow rate, and h_i (m) the level in tank N_i.

The flow response of the valve V_i is described by the equation

$$q_i = c_{Li} h_i + c_{Ti} \sqrt{h_i} \tag{2}$$

where c_{Li} ($m^2\ s^{-1}$) is laminar component of the conduction of the valve V_i, c_{Ti} ($m^{2.5}\ s^{-1}$) is the turbulent component of the conduction of the valve V_i.

We can write the mathematical model of the third-order system without interaction

$$\frac{dh_1}{dt} = \frac{1}{F}(q_0 - c_{L1}h_1 - c_{T1}\sqrt{h_1}) \tag{3}$$

$$\frac{dh_2}{dt} = \frac{1}{F}(c_{L1}h_1 + c_{T1}\sqrt{h_1} - c_{L2}h_2 - c_{T2}\sqrt{h_2}) \tag{4}$$

$$\frac{dh_3}{dt} = \frac{1}{F}(c_{L2}h_2 + c_{T2}\sqrt{h_2} - c_{L3}h_3 - c_{T3}\sqrt{h_3}) \tag{5}$$

RESULTS AND DISCUSSION

To choose a control algorithm based on the reference model for the laboratory tank system we have to take into account several facts:

- The system is nonlinear;
- The time constant is large;
- The system exhibits time delay.

Model reference adaptive control (MRAC) is one of the approaches to adaptive control [3]. We have chosen MRAC for its simplicity and robust quality. The goal of MRAC is to suggest a control loop to achieve asymptotic stability of the whole loop and to control the error convergence to zero. One of the advantages of MRAC is that it is adaptive control without online identification of system parameters. The control feedback loop of the plant contains a parallel reference model R_M. MRAC usually works with unknown plant and its behaviour is prescribed by the behaviour of the reference model. The outputs from control plant and reference model are measured and compared. The difference between the model reference output and the controlled output (the model error) is utilizing in particular adaptive control laws. These adaptive laws are adjusting the controller so that the conditions of Lyapunov stability theory are fulfilled. The form of the Lyapunov functions is described for example in [3] and [4]. As shown in Fig. 2, in the direct MRAC the control input u is obtained only on the basis of the output information y of the plant. The following equations describe the control problem of the first-order system

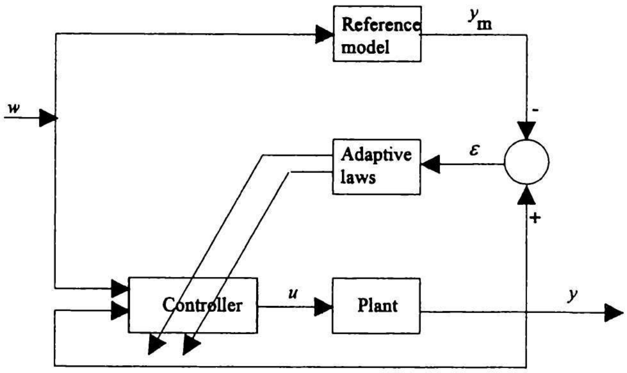

Fig. 2. MRAC of the first-order system.

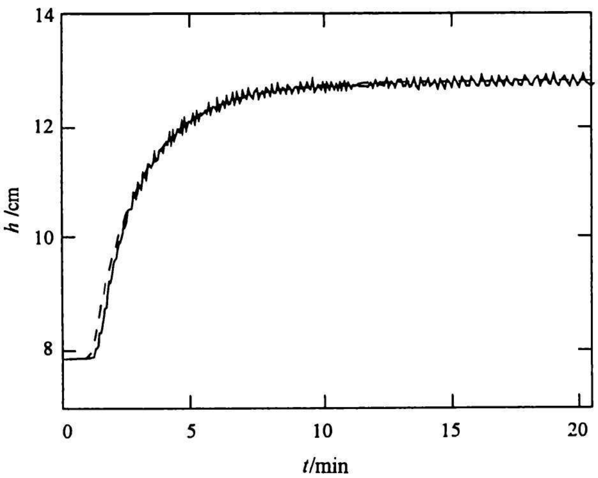

Fig. 3. The control of the first-order system. - - - - Reference model, —— plant.

plant $$\frac{\mathrm{d}y(t)}{\mathrm{d}t} = -ay(t) + bu(t) \qquad (6)$$

reference model $$\frac{\mathrm{d}y_\mathrm{m}(t)}{\mathrm{d}t} = -a_\mathrm{m}y_\mathrm{m}(t) + b_\mathrm{m}w(t) \qquad (7)$$

control law $$u(t) = r_0(t)w(t) + g_0(t)y(t) \qquad (8)$$

adaptive laws $$\frac{\mathrm{d}r_0(t)}{\mathrm{d}t} = -\gamma_1\varepsilon(t)w(t) \qquad (9)$$

$$\frac{\mathrm{d}g_0(t)}{\mathrm{d}t} = -\gamma_2\varepsilon(t)y(t) \qquad (10)$$

error $$\varepsilon(t) = y(t) - y_\mathrm{m}(t) \qquad (11)$$

$$\lim_{t\to\infty} \varepsilon(t) = 0 \qquad (12)$$

where w is the set point, y_m output of reference model, y output of plant, g_0, r_0 feedback and feedforward controller parameters, γ_1, γ_2 adaptive gains, ε adaptive error.

It is very important to choose the reference model and adaptive gain to insure stability of the system. The time constant of the reference model and the time constant of the controlled plant have to be similar. Fig. 3 shows the control of the first-order system.

MRAC of higher-order systems uses derivatives if inputs and outputs to obtain a control law. The derivatives are not directly measurable in general, so that they have to be generated. Good results can be obtained by applying steady-state filters for both, input and output variables.

The assumptions for the control design of the second- and third-order systems are:

- The method MRAC is designed for the relative degree r (r = deg (A) − deg (B), A and B are polynomials of the plant transfer function);
- Reference model is of the second order;
- Estimation of derivative of the input and output signals is obtained by linear first-order filters.

The final form of the control algorithm for the relative degree $r = 1$ (for the system of the second order) is

reference model $$\frac{\mathrm{d}^2y_\mathrm{m}(t)}{\mathrm{d}t^2} = -a_{\mathrm{m}2}\frac{\mathrm{d}y_\mathrm{m}(t)}{\mathrm{d}t} - a_{\mathrm{m}1}y_\mathrm{m}(t) + b_{\mathrm{m}0}u(t) \qquad (13)$$

control law $$u(t) = k(t)w(t) + \theta_1^\mathrm{T}(t)\omega_1(t) + \theta_0(t)y(t) + \theta_2^\mathrm{T}(t)\omega_2(t) \qquad (14)$$

adaptive laws

$$\frac{dk(t)}{dt} = -\gamma \varepsilon(t) w(t) \quad (15)$$

$$\frac{d\theta_1(t)}{dt} = -\gamma \varepsilon(t) \omega_1(t) \quad (16)$$

$$\frac{d\theta_0(t)}{dt} = -\gamma \varepsilon(t) y(t) \quad (17)$$

$$\frac{d\theta_2(t)}{dt} = -\gamma \varepsilon(t) \omega_2(t) \quad (18)$$

where a_{mi}, b_{mi} are parameters of the reference model ($a_{mi} > 0$ and $b_{mi} > 0$), w is the set point, y_m output of the reference model, y output of plant, γ adaptive gain, ω_1, ω_2 estimated derivatives (outputs of linear filters), θ_i parameters of controller, $\varepsilon(t)$ error (obtained as for the first-order MRAC).

The control law for the relative degree $r = 2$ (for the third-order system) is modified as follows

$$u(t) = k(t)w(t) + \theta_1^{T}(t)\omega_1(t) + \theta_0(t)y(t) + \\ + \theta_2^{T}(t)\omega_2(t) - \varepsilon\overline{\omega}^{T}\overline{\omega} \quad (19)$$

where $\overline{\omega}$ is the filtered vector of the steady-state values ω.

The block diagram for the MRAC of the relative degree $r = 2$ is shown in Fig. 4. For the relative degree $r = 1$ the same diagram can be used without the block of the transfer function S. The control of the second- and third-order systems is shown in Figs. 5 and 6, respectively.

CONCLUSION

In this paper algorithms of model reference adaptive control of a laboratory tank system have been presented. The control algorithms were carried out within the Matlab and Simulink software environment on a PC 486. Connection between PC and laboratory model was realized by input-output signal converters PCL 812PG and PCL 728 (ADVANTECH) using Matlab Real Time Toolbox. The obtained control results show feasible performance of the control system especially in cases of the first- and second-order systems. The time constant of the given reference model has the major influence on the control quality. The good results are obtained when the time constant of the reference model is near to the time constant of the plant. The choice of the adaptive gains is also very important. These gains can be designed by simulation and experiments.

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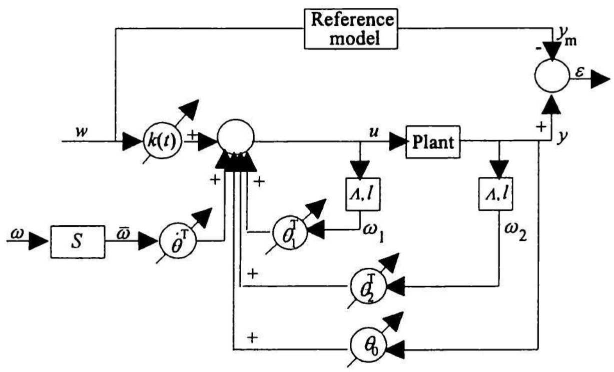

Fig. 4. Control structure for the MRAC of the relative degree $r = 2$. $\dot{\theta}$ – the parameter vector specified by an adaptive law, Λ, l – parameters of the first-order filters.

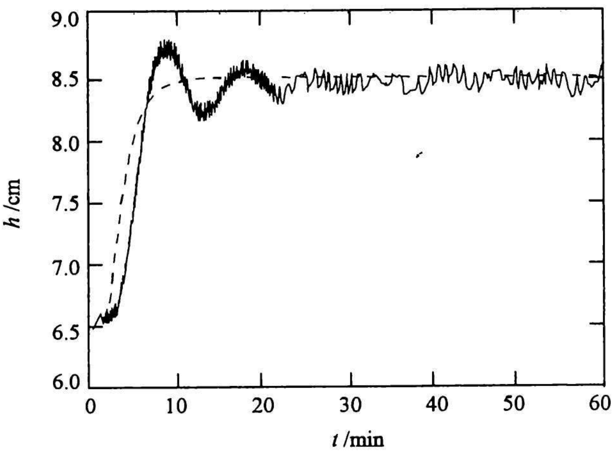

Fig. 5. The control of the second-order system. – – – – Reference model, —— plant.

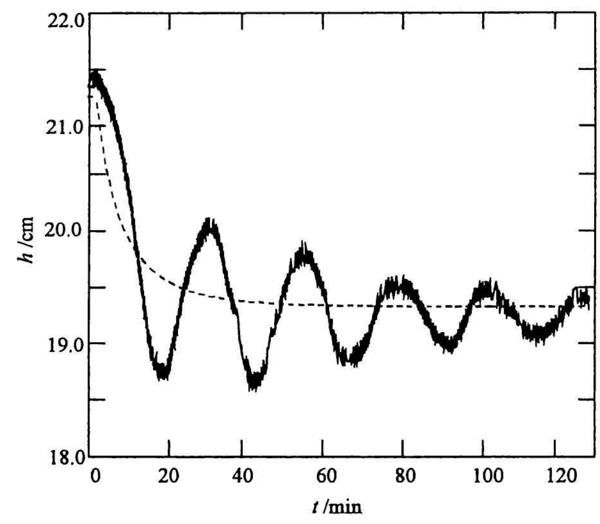

Fig. 6. The control of the third-order system. – – – – Reference model, —— plant.

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