Thermodynamic Equilibrium between Melt and Crystalline Phase of a Compound A_qB_r with Dystectic Melting Point IV. Application to the Systems NaF—NaMgF₃ and KF—KMgF₃

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The LeChatelier—Shreder equation describing the equilibrium between crystalline phase and melt in the system A—AB when substance AB partially dissociates upon melting was applied in the systems NaF—NaMgF₃ and KF—KMgF₃. The dissociation enthalpy, the heat of fusion at dystectic temperature of fusion, and the degree of dissociation of compounds NaMgF₃ and KMgF₃, formed in the melt, were calculated without using any fictive quantity. The dissociation degrees of NaMgF₃ and KMgF₃ have been found to be $\alpha_+(NaMgF_3) = 0.23$ and $\alpha_+(KMgF_3) = 0.11$, respectively.

In the previous paper [1] the LeChatelier—Shreder equation describing the equilibrium between crystalline phase and melt in the system $\mathbf{A} - \mathbf{A}_{q} \mathbf{B}_{r}$ when the substance $A_q B_r$ partially dissociates upon melting was derived without assuming any hypothetical equilibrium states and without the occurrence of any hypothetical thermodynamic quantities. To fit both the phase and the chemical equilibrium in the system $\mathbf{A} - \mathbf{A}_{\sigma} \mathbf{B}_{r}$ an original method for the calculation of the dissociation enthalpy, the heat of fusion at dystectic temperature of fusion, and the degree of dissociation of the compound $A_q B_r$ in the melt was proposed. However, there are not three components in the system, but according to the Gibbs phase law only two. The weighted-in substances A and $A_{\sigma}B_{r}$ are thus components and the present substances at equilibrium, A, B, $A_{a}B_{r}$, are called constituents. In order to distinguish between them the components will be in the subsequent text denoted as bold characters, the constituents of the quasi-ternary system as plain ones.

In the present paper the proposed method was applied to the systems NaF— $NaMgF_3$ and KF— $KMgF_3$, which are subsystems of the binary systems NaF— MgF_3 and KF— MgF_2 , in which the additive compounds $NaMgF_3$ and $KMgF_3$, respectively, are formed. In this particular case q = r = 1.

The phase diagram of the system $NaF - MgF_3$ was measured by *Bergman* and *Dergunov* [2]. These authors found out that a new congruently melting compound $NaMgF_3$ with the melting point of 1030 °C is formed in this system. The coordinates of the individual eutectic points are: 23.2 mole % MgF_3 , 830 °C and 62 mole % MgF_2 , 1000 °C.

The phase diagram of the system $KF-MgF_3$ was measured by *DeVries* and *Roy* [3]. Two new com-

pounds are formed in this system. The compound $\mathbf{KMgF_3}$ melts congruently at the temperature of 1070 °C, while the compound $\mathbf{K_2MgF_4}$ melts incongruently, with the temperature of peritectic decomposition 846 °C. The authors observed a relatively wide range of solid solutions of \mathbf{KF} in $\mathbf{KMgF_3}$ with the possible transformation of α to β solid solution near the melting point of $\mathbf{KMgF_3}$.

The enthalpies of fusion of both intermediate congruently melting compounds $NaMgF_3$ and $KMgF_3$ were measured by *Adamkovičová et al.* [4].

The thermodynamic analysis of the systems MF— $MMgF_3$ (M = Na, K), in which the component $MMgF_3$ thermally dissociates at melting according to the reaction

$$AB \Leftrightarrow A + B$$
 (l) (A)

is based on the fundamental relations derived in our previous papers [5, 6].

$$\Psi = \frac{T T_{\text{fus}}(\mathbf{AB})}{T - T_{\text{fus}}(\mathbf{AB})} \ln \frac{x (AB, T)}{x_{+} (AB, T)}$$
(1)

For limiting values of Ψ and its derivative the following relations are valid

$$\lim_{\substack{z_{w} \to 1 \\ T \to T_{fus}}} \Psi = \frac{\Delta_{fus, exp} H(AB, T_{fus})}{R}$$
(2)

$$\lim_{\substack{xw \to 1\\ T \to T_{tws}}} \frac{\mathrm{d}\Psi}{\mathrm{d}x_{w}} = 0 \tag{3}$$

For the equilibrium mole fractions of individual constituents the following relations, obtained by means of the weighted-in equilibrium mole fraction, $x_{\rm w}$, and the dissociation degree of reaction (A), are valid

$$x(AB) = \frac{(1 - \alpha_{(A)}(T))x_{\mathbf{w}}}{x_{\mathbf{w}}\alpha_{(A)}(T) + 1}$$
(4)

$$x(A) = \frac{1 - (1 - \alpha_{(A)}(T))x_{w}}{x_{w}\alpha_{(A)}(T) + 1}$$
(5)

$$x(\mathbf{B}) = \frac{x_{\mathbf{w}}\alpha_{(A)}(T)}{x_{\mathbf{w}}\alpha_{(A)}(T) + 1} \tag{6}$$

For the equilibrium constant of reaction (A) we then get

$$K_{(A),x} = \frac{x(A)x(B)}{x(AB)} = \frac{x_{+}(A)x_{+}(B)}{x_{+}(AB)} = K_{(A),x,+}$$
(7)

For ideal solutions eqn (7) holds exactly, for real solutions its validity is assumed for limiting compositions $(x_{\mathbf{w}} \rightarrow 1, T \rightarrow T_{\mathsf{fus}}(\mathbf{AB})).$

In the system KF-KMgF₃ solid solutions of KF in the dystectically melting component $KMgF_3$ are formed [3]. From the course of the solidus curve it follows, however, that in the solid solution no dissociation of KMgF3 takes place. This must be taken into account also in the calculation.

The equilibrium between the liquid and solid solutions is described by the equation

$$\mu(\mathbf{AB}, \mathrm{ss}, x_{\mathbf{w}, \mathbf{s}}, T) = \mu(\mathbf{AB}, \mathrm{ls}, x_{\mathbf{w}, \mathbf{l}}, T) \qquad (8)$$

where $x_{w,s}$ and $x_{w,l}$ are the equilibrium mole fractions of AB in the coexisting solid and liquid solution, respectively.

Assuming the ideal behaviour of the solid solution and inserting for $\mu(AB)_i$ into eqn (8), we get the relation

$$\mu_{+}(\mathbf{AB}, s, T) + RT \ln x_{\mathbf{w}, s}(\mathbf{AB}, ss, T) =$$

$$= \mu_{+}(\mathbf{AB}, \mathbf{l}, T) + RT \ln \frac{a(\mathbf{AB}, \mathbf{ls}, x_{\mathbf{w}, \mathbf{l}}, T)}{a_{+}(\mathbf{AB}, \mathbf{ls}, T)} \qquad (9)$$

Using eqn (9), relations (1-3) transform to the form [5, 6]

$$\Psi_{ss} = \frac{TT_{fus}(\mathbf{AB})}{T - T_{fus}(\mathbf{AB})} \ln \frac{x (AB, ls, x_{w,l}, T)}{x_{+} (AB, ls, T) x_{w,s} (AB, ss, T)}$$
(10)
$$\lim_{\substack{x_{w,l} \to 1 \\ x_{w,s} \to 1 \\ T \to T_{fus}}} \Psi_{ss} = \frac{\Delta_{fus, exp} H(AB, T_{fus})}{R}$$
(11)

$$\rightarrow T_{fus}$$

$$\lim_{\substack{x_{\mathbf{w},l} \to 1 \\ x_{\mathbf{w},\bullet} \to 1 \\ T \to T_{\text{fus}}}} \frac{\mathrm{d}\Psi_{\text{ss}}}{\mathrm{d}x_{\mathbf{w},l}} = 0 \tag{12}$$

RESULTS AND DISCUSSION

The goal of the calculation in the systems NaF-NaMgF₃ and KF-KMgF₃ was to find the values of $\alpha_{(A),+,i}(T_{\mathrm{fus},i})$ to which correspond the limiting values of Ψ and Ψ_{ss} fulfilling eqns (2) and (11) for the calorimetrically measured enthalpies of fusion of NaMgF3 and $\mathbf{KMgF_3}$ [4]. With regard to the limiting conditions, the temperature dependence of $\alpha_{(A),+,i}$ was neglected in the calculation of Ψ and Ψ_{ss} . The functions Ψ and Ψ_{ss} were determined for three equilibrium values of $T_{i,j}$ on the corresponding liquidus, resp. solidus curves taken from [2, 3]. The liquidus curve of NaMgF₃ in the system NaF-NaMgF₃ is shown in Fig. 1 and that of KMgF₃ in the system KF-KMgF₃ in Fig. 2.

The functions Ψ and Ψ_{ss} were extrapolated for $T_{i,i} \rightarrow T_{\text{fus},i}$ using the regression dependences of the form

$$\Psi_{i,j} = A_{i,j} + B_{i,j} (1 - x_{w,l})^{C_{i,j}}$$
(13)

fulfilling the conditions (2) and (11), where

$$A_{i,j} = \frac{\Delta_{\text{fus}} H_{i,j}}{R} \tag{14}$$

For the chosen values of $\alpha_{(A),+,i,j}(T_{\text{fus},i})$ the corresponding values of $\Delta_{fus} H_{i,j}$ were calculated from the limiting relations (2) and (11). From the empirical relation

$$\varphi_i(\Delta_{\mathrm{fus}}H_{i,j},\alpha_{(A),+,i}) = 0 \tag{15}$$

the dissociation degrees $\alpha_{(A),+,i}$ were calculated for the calorimetrically measured value of $\Delta_{fus,exp} H_i$. The results of calculation are summarized in Table 1.

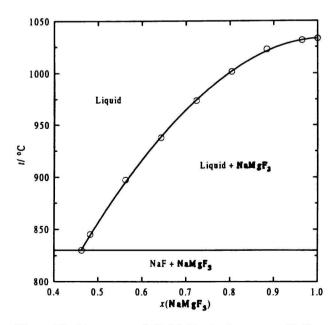


Fig. 1. Liquidus curve of NaMgF3 in the system NaF-NaMgF₃

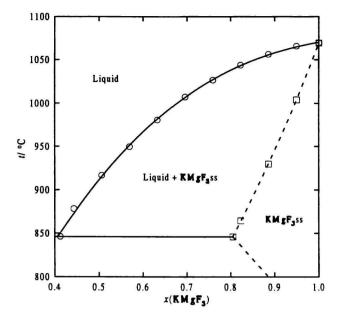


Fig. 2. Liquidus curve of KMgF₃ in the system KF-KMgF₃.

Table 1. Experimentally Determined Values of the Enthalpies of Fusion and the Calculated Values of the Dissociation Degrees of the Compounds NaMgF₃ and KMgF₃

Component	$\frac{\Delta_{\rm fus,exp} H({\bf AB},T_{\rm fus}) \ [5]}{\rm J \ mol^{-1}}$	$\alpha_{(A),+,i}(T_{\mathrm{fus},i})$
NaMgF ₃	66 000 ± 4 000	0.233
KMgF ₃	96 000 ± 5 000	0.112

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SYMBOLS

- A, B, AB components of the binary systems
- A, B, AB constituents, *i.e.* the "components" of the quasi-ternary system
- x_w weighted-in mole fraction of **AB** in the system **A**—**AB**
- x(AB,T) equilibrium mole fraction of AB in the mixture at temperature T
- $T_{\rm fus} ({\bf AB})$ temperature of fusion of ${\bf AB}$
- $\Delta_{\text{fus,exp}} H(\mathbf{AB}, T_{\text{fus}})$ experimentally determined enthalpy of fusion of \mathbf{AB} at its temperature of fusion
- $\Delta_{fus}H_i$ calculated enthalpy of fusion of AB
- $\alpha_{(A)}(T)$ dissociation degree of AB in reaction (A) at temperature T
- $K_{(A),x}(T)$ equilibrium constant of reaction (A) related to mole fractions in the mixture at temperature T
- $\mu(AB)$ chemical potential of AB
- a(AB) activity of AB in the mixture
- $A_{i,j}, B_{i,j}, C_{i,j}$ constants in empirical equation l liquid
- ls liquid solution
- s solid
- ss solid solution
- + related to pure **AB**

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